Optimization

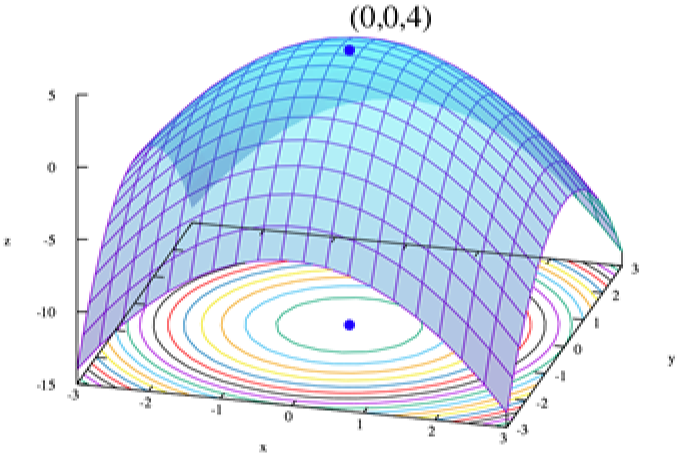
Jasper, Jordan, Lucy

Overview

- Optimization Under Uncertainty
- Pareto Optimization (Multiple Objectives)
- Structural Optimization
- Combinatorial Optimization
- Integer Programming
- Constraint Programming
- Linear Programming
- Convex Optimization
- Dynamic Programming
- Questions

Introduction

Optimization is finding the best element, value or result based on certain constraints or criterion. It can be found in every field from engineering to economics, and most disciplines involving numbers and situations. It even crops up in day-to-day life.



Stochastic Programming, Chance-Constrained Programming & Robust Optimization

Uncertainty / Randomness

Accounting for Uncertainty

stochastic programming

chance-constrained programming

robust optimization

Approaches

These are different ways of approaching optimization with uncertainty in the picture.

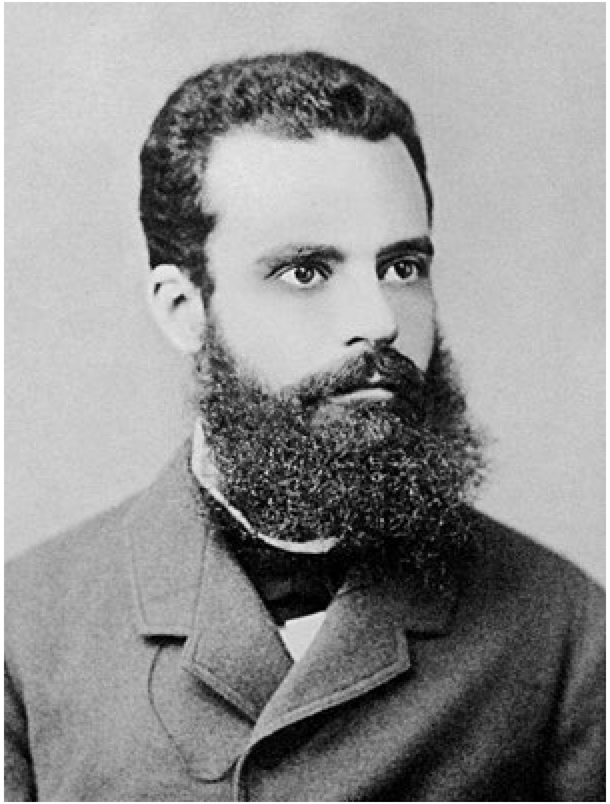
Stochastic programming	Chance-constrained programming	Robust optimization

- "risk-neutral"
- probability distributions
    - sometimes unknown
- expected values
	- risk-averse
	- some parameters are allowed to be violated by chance
	    - as long as overall is kept
	- guarantees some quality
	    - chance-constrained
		- worst-case scenario
		- no assumptions
		- uncertainty sets
		- guarantees some quality

Pareto Optimization

Optimizing Multiple Objectives

Better for all



Pareto optimality / efficiency

- nothing can make a situation better in all respects
- economics, engineering, science, math, game theory, everyday

Example Scenario 1

Person A

Apples	+1 like point
Bananas	+0 like points

Person B

Apples	+0 like points
Bananas	+1 like point

10 apples

10 bananas

Optimal state:

A: 10 apples

B: 10 bananas

Example Scenario 2

Person A	Apples	+2 like points	Person B	Apples	+1 like point
	Bananas	+1 like point		Bananas	+2 like points

10 apples

10 bananas

Optimal states:

A: 10 apples

B: 10 bananas

A: 10 apples, 10 bananas

B:

A:

B: 10 apples, 10 bananas

Production -Possibility

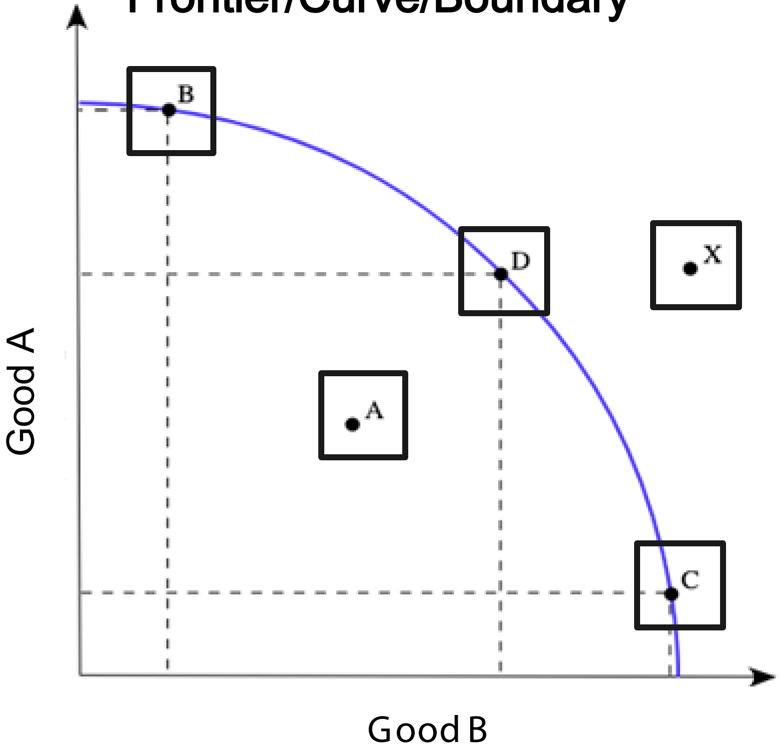
The PPF/PPC/PPB can be used to measure Pareto optimality.

- possible quantities of two goods
- used in economics

$ay + bx = n$ is a line, not a curve.

Law of Diminishing Returns

Production -Possibility Frontier/Curve/Boundary

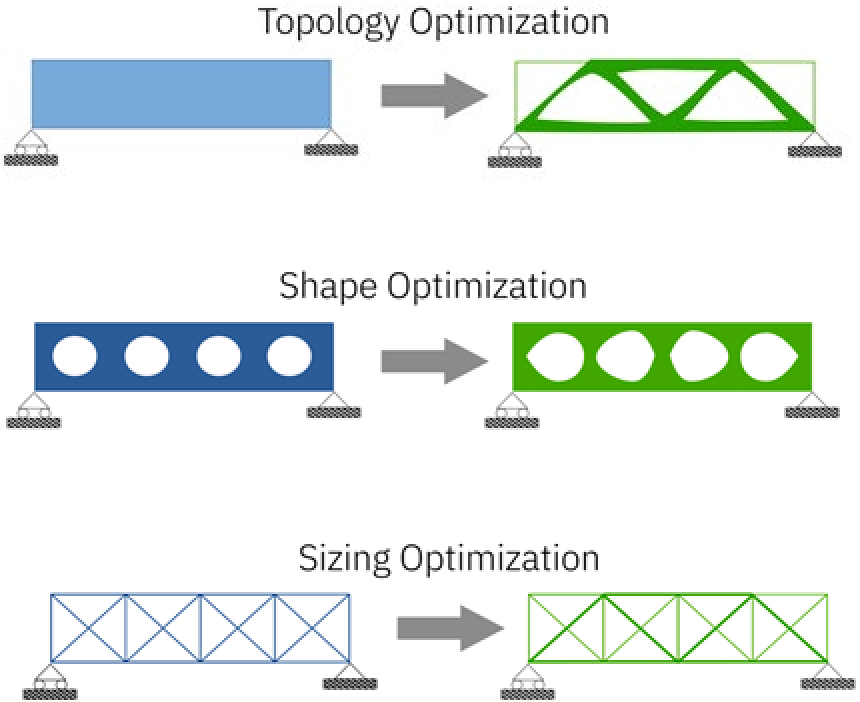


Structural Optimization

Shape, Topology, Size

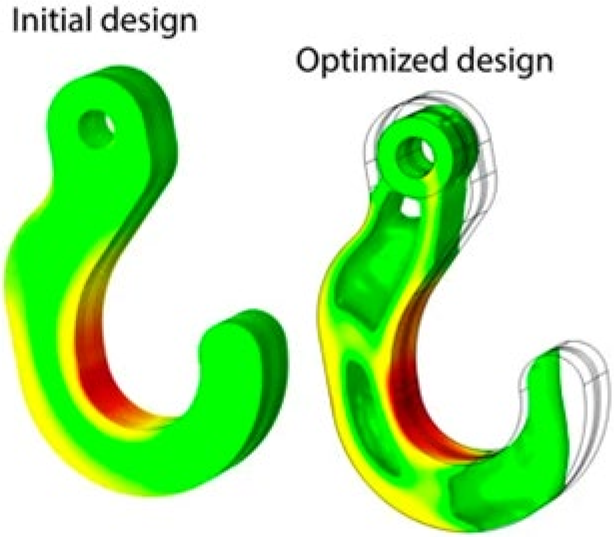
Structural Optimization

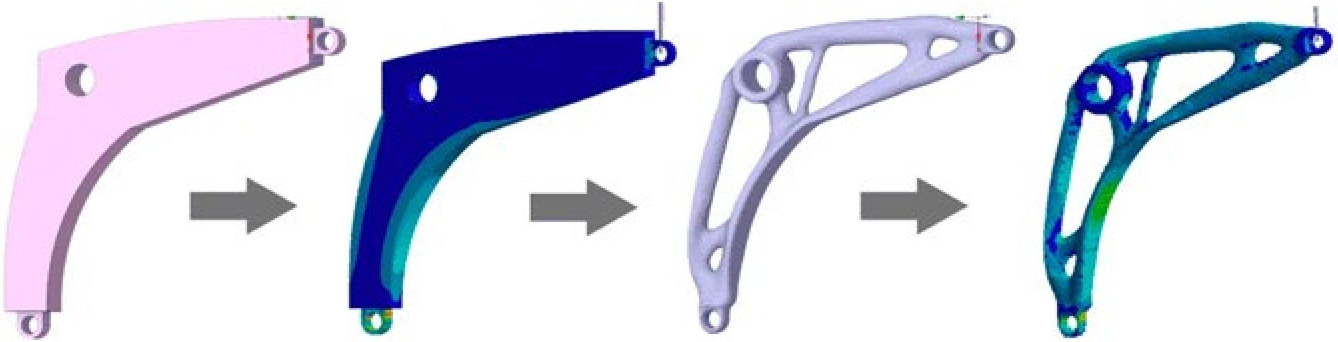
- doing more with less
- structures withstanding forces
- nature
- manufacturing
- 3d structures



Topology Optimization

- takes any shape
- basic, conceptual
- connectivity of holes
- cuts unnecessary materials

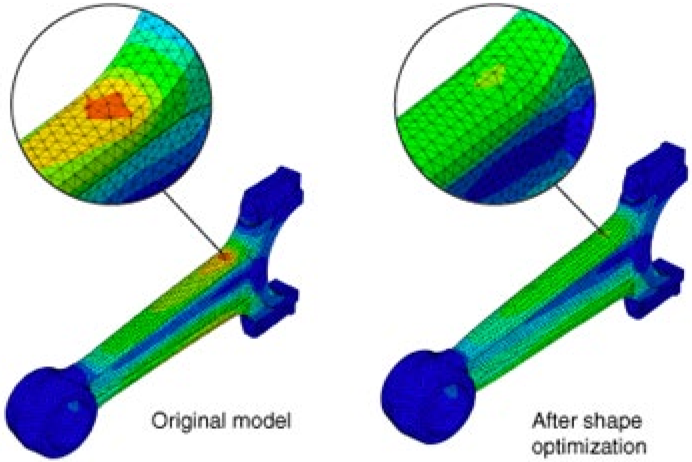




Shape Optimization

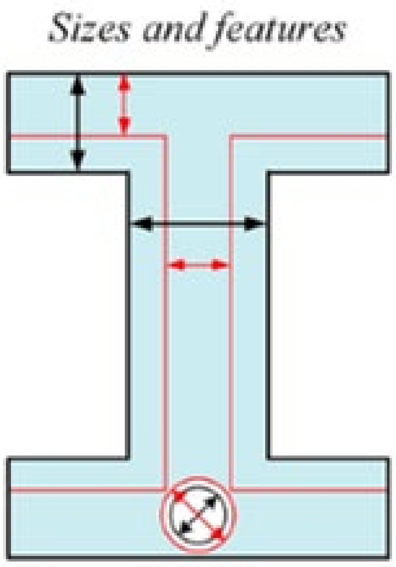
- minimizing surface area or volume
- more reliable
- redesigns areas of stress

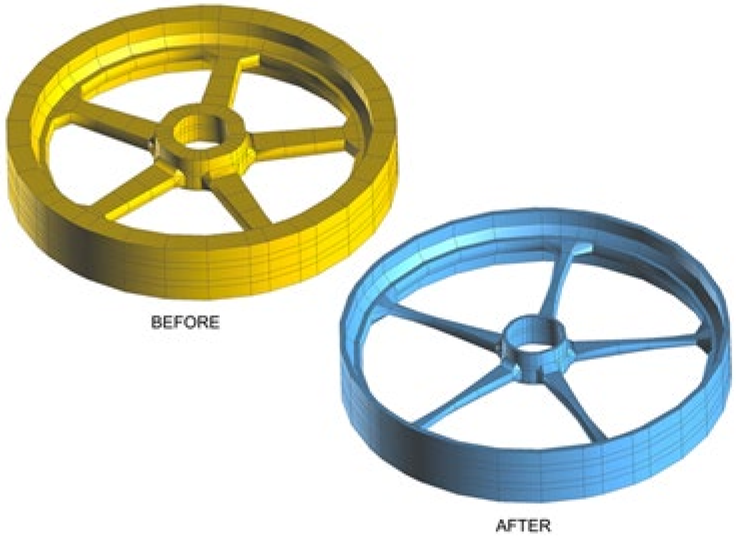
High stress concentrations



Size Optimization

- final design phase
- thickness, ply shapes
- stacking sequence



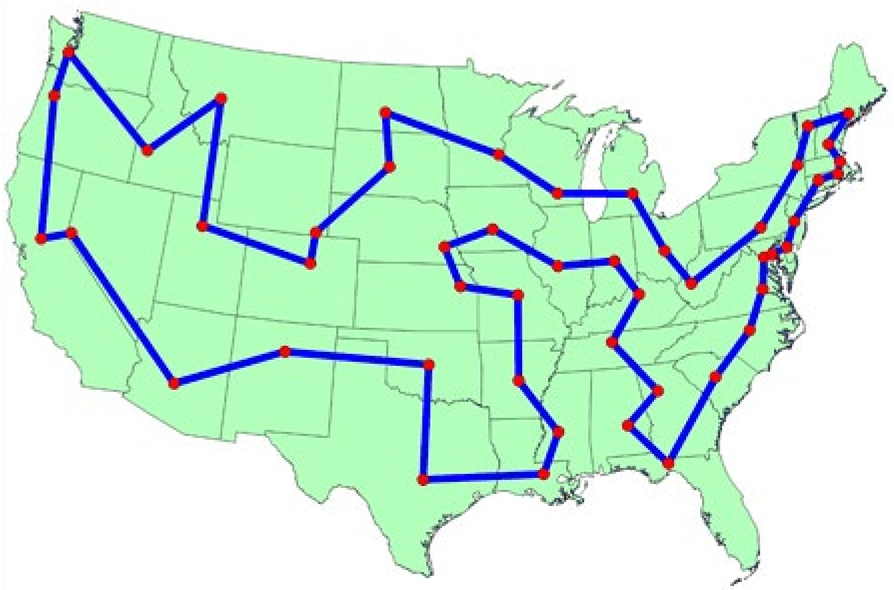


Combinatorial Optimization

Finding the best option

Combinatorial Optimization

- Asks you to find the optimal “thing” in a discrete set
- Cost effectiveness
- Logistics, jobs, geoengineering, planning routes, prioritizing

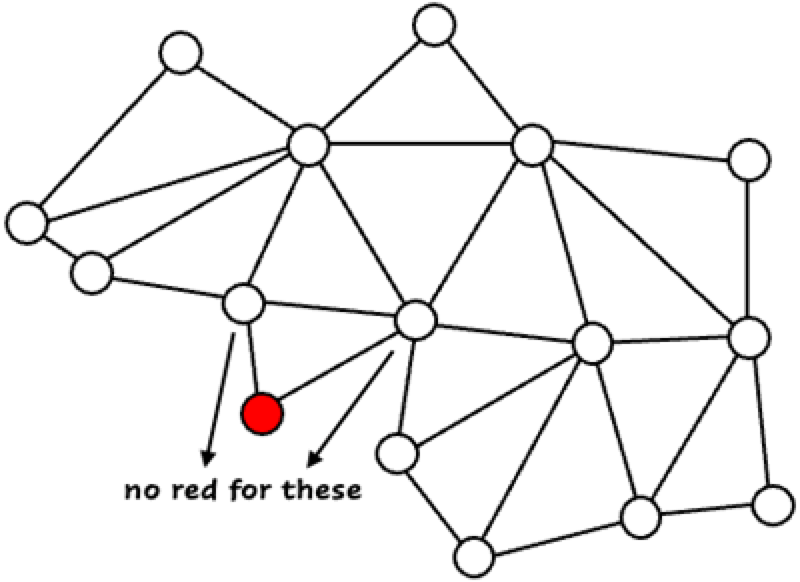


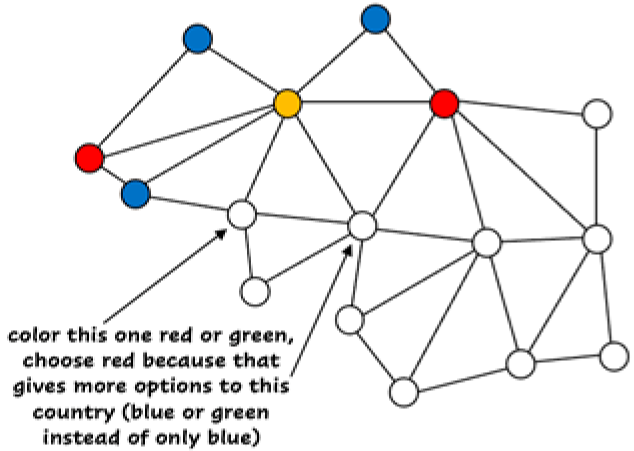
Integer Programming

- Optimizes integers
- Often hard to find the answer to, as there are many solutions
- Used for scheduling, routing, etc.

Constraint Programming

- Helps you find a solution or the best solution based on certain constraints
- Verifying, planning, scheduling





Linear Programming

History

- Dates to 1827 when Jean-Baptiste Joseph Fourier published a method to solve them
- Around 1939, Leonid Kantorovich and T.C. Koopmans formulated linear programming problems.
- They would win the Nobel Prize in 1975
- Frank Lauren Hitchcock also helped develop linear programming, but died in 1957



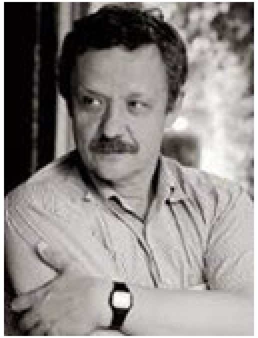
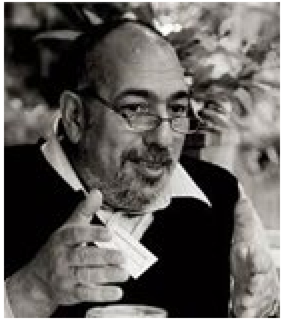

Linear Programming

- Also known as Linear Optimization
- Minimizes or Maximizes a function
- Subject to linear constraints, which create the feasible region
- An objective function is created

Convex Optimization

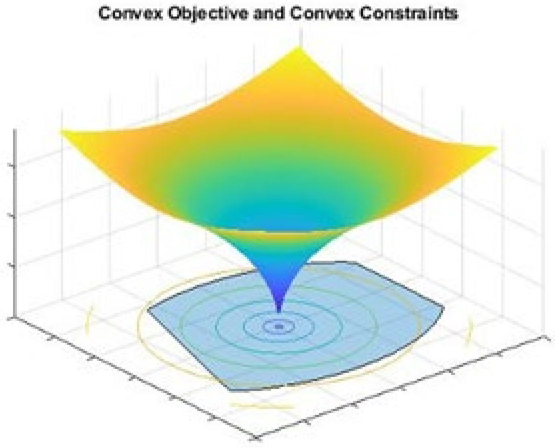
History

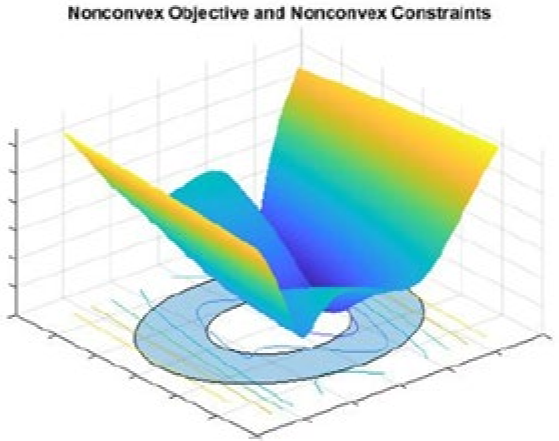
- Convex analysis theory developed from 1900 to 1970
- 1947, Dantzig's simplex algorithm was developed
- In the 1980s and 90s, polynomial-time interior -point methods developed for convex optimization
- Before 1990s, was used for business
- After 1990s, had more applications to engineering and machine learning

Convex Optimization

- Similar to linear programming
- Constraints are convex instead of linear
- Looks like a bowl
- Minimization is convex, maximization is concave
- Solution is the global minimum, meaning it is the only answer





Dynamic Programming

History

- Term coined by Richard Bellman
- Described as solving problems where one need to find the ebay decision one after another
- In 1953, term was refined as nesting smaller problems inside larger decisions

Dynamic Programming

- Dynamic programming breaks problem into smaller subproblems
- Help solve recursive problems with memoization
- If solution is needed, memoization first sees if that problem has already been solved or not
- If solution is not there, the problem will have to be solved and then will be added to the table

```
F(0) = 0
F(1) = 1
F(2) = F(1) + F(0)
F(3) = F(2) + F(1)
F(4) = F(3) + F(2)
```

Questions

Question 1

Pareto Optimization

Revisiting the two-person society...

As a reminder, there are only two people, Person A and Person B. Person A likes apples and Person B likes bananas.

However, in this situation, Person A gets **2 like points from apples** and **1 like point from bananas**. Likewise, Person B gets **2 like points from bananas** and **1 like point from apples**.

If there are 10 of each good, how many Pareto optimal states are there now?
bonus: what are they?

A. 3

B. 21

C. 1

D. 12

E. 23

Solution 1

Pareto Optimization

Person A: 10 apples
Person B: 10 bananas

Person A: 10 apples 10 bananas
Person B:

Person A:
Person B: 10 apples 10 bananas

x: {1, 2, 3, 4, 5, 6, 7, 8, 9}

Person A: 10 apples x bananas
Person B: 10-x bananas

Person A: 10-x apples
Person B: x apples 10 bananas

1 +2 +9 +9 =21 Pareto optimal states.

A. 3

B. 21

C. 1

D. 12

E. 23

Question 2

Linear Programming

Sara wants to start a lemonade stand and wants to determine the optimal number of small and large cups of lemonade to sell to maximize her profit. She has limited resources and the following constraints apply:

Each small cup of lemonade requires 1 lemon and 0.5 cups of sugar.
Each large cup of lemonade requires 2 lemons and 1 cup of sugar.
Sara has 6 lemons and 4 cups of sugar available.

Sara can sell each small cup with a profit of $1.50 and each large cup with a profit of $2.50.

Which combination of small and large cups should Sara sell to maximize her profit?

A. 4 small and 1 big

B. 6 small and 0 big

C. 0 small and 3 big

D. 2 small and 2 big

Solution 2

Linear Programming

Each small cup of lemonade requires 1 lemon and 0.5 cups of sugar.
Each large cup of lemonade requires 2 lemons and 1 cup of sugar.
Sara has 6 lemons and 4 cups of sugar available.

Sara can sell each small cup with a profit of $1.50 and each large cup with a profit of $2.50.

A = $8.50
B = $9
C = $7.50
D = $8

Therefore, B is optimal.

A. 4 small and 1 big

B. 6 small and 0 big

C. 0 small and 3 big

D. 2 small and 2 big

Question 3

Stochastic Programming

You have a standard deck of cards but without the faces. (So, only the numbers, making 40 cards in total.) You're playing a game with your friend where you try to get the numbers on your cards to add up to exactly or under 21. You currently have a 9 and a 7. Your friend has a 3 and a 5, and it's your turn. You and your friend have already played 3 rounds of this game, and the total of the 20 cards already played is 124. If you get over 21, you lose. *Should* you draw another card?

Remember: Stochastic programming in this case is optimizing the expected value.

Draw a card

Keep it

Solution 3

Stochastic Programming

There are 16 cards left, and you have a 16. If you were to draw a card, the average must be 5 or less.

The 40 cards sum to 220. The cards that you and your friend have currently sum to 24, plus the 124 already played.

220 - (124 + 24) = 72, and you have 16 cards left.

72 / 16 = 4.5, so you’re just slightly more likely to get under 21.

Draw a card

Keep it

Question 4

Convex Optimization
Short Answer

What are the differences and similarities between convex optimization and linear programming?

Solution 4

Convex Optimization
Short Answer

Similarities

- Objective functions
- Constraints
- Feasible Region
- Solved with algorithms
- Linear programming is a special case of convex optimization

Differences

- Linear function instead of convex function
- Linear constraints instead of convex constraints
- The solutions for linear problems are more discrete, while convex optimization is more continuous
- Algorithms are different because of the special properties of lines
- Convex optimization is non-linear

Conclusion