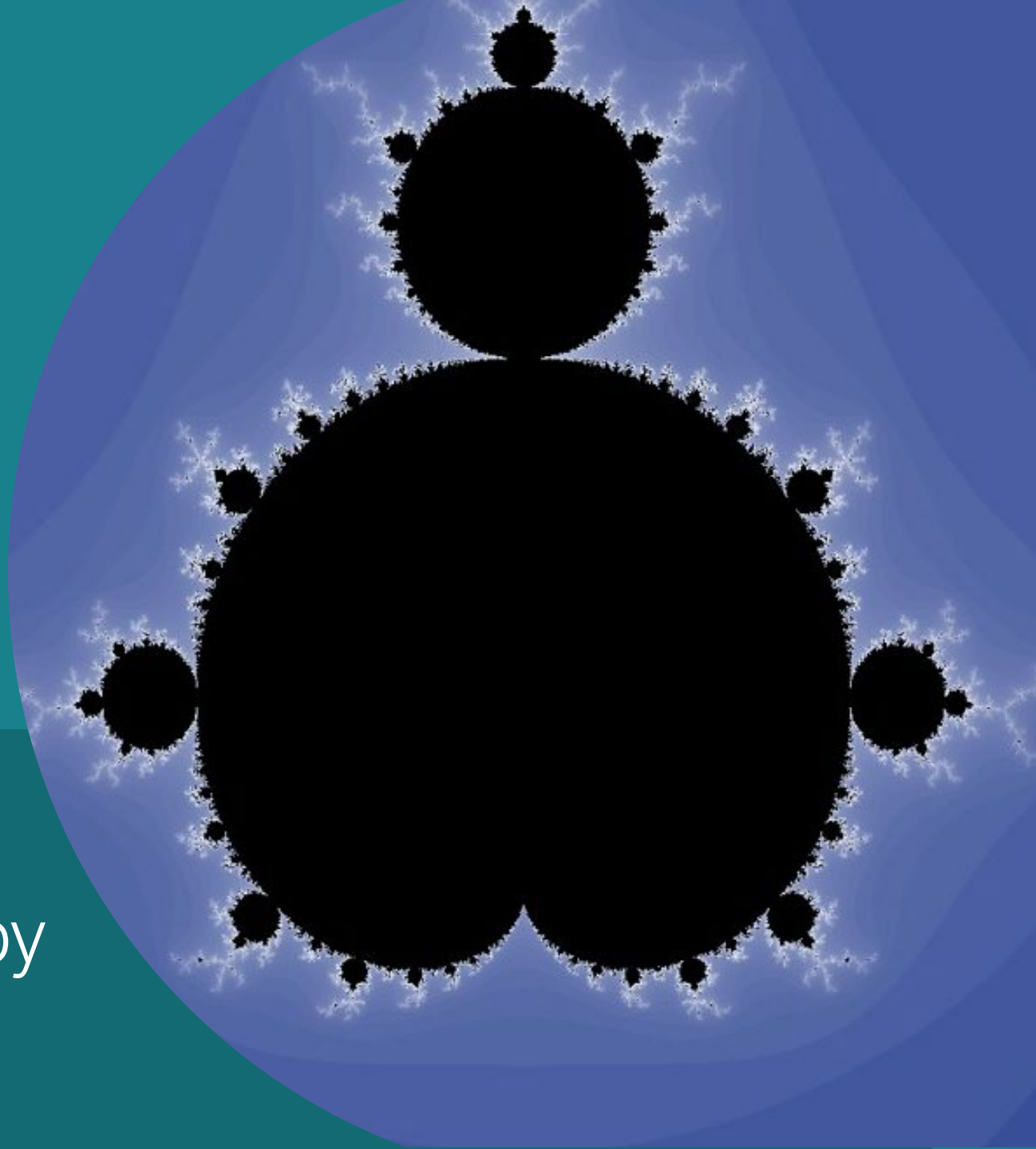
Fractals

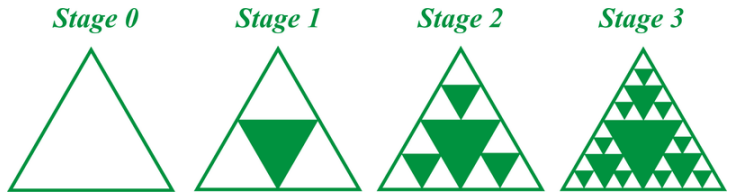
Infinitely Complex Shapes



By Shashwath, Bryan, and Toby

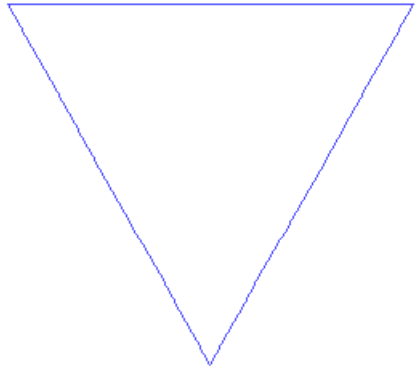
Sierpinski Triangle

- The Sierpinski triangle is a self-similar fractal. It consists of an equilateral triangle, with smaller equilateral triangles recursively removed from its remaining area. Which is named after the Polish mathematician Waclaw Sierpinski
- How to construct a Sierpinski triangle
  - Step 1: Draw a triangle
  - Step 2: Draw a triangle inside of the triangle
  - Step 3: Draw a triangle inside those triangles, but not the one in the middle
  - Step 4: Repeat steps 2-3



- Where *n* is the desired iteration step,
  - The perimeter's formula is $P_n = 96n$
  - The area's formula is $A_n = \sqrt{34} \cdot 34n$
- Where *n* is the number of self-similar pieces, *m* is the magnification factor, and *d* is the dimension.
  - The fractal dimension of the Sierpinski triangle is $n = m^d$

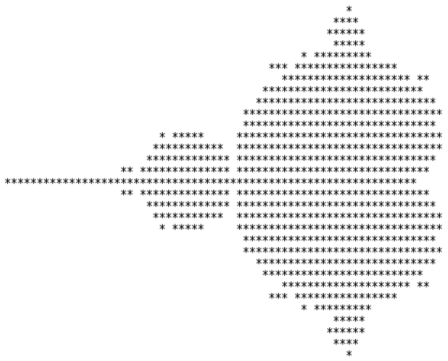
Koch Snowflake



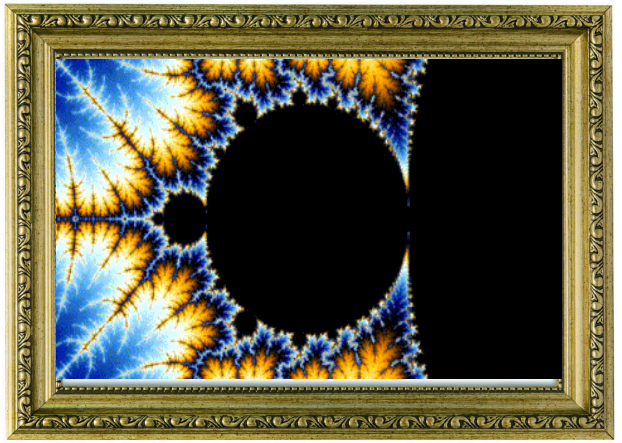
- The Koch snowflake was one of the first fractals to be discovered.
- It is constructed using equilateral triangles and the following method:
  1. Divide the line segment into three segments of equal length.
  2. Draw an equilateral triangle that has the middle segment from step 1 as its base and points outward.
  3. Remove the line segment that is the base of the triangle from step 2.
- Using complex mathematical formulas, we can derive the following:
  If the original equilateral triangle has sides of length s, the perimeter of the snowflake after n iterations is $3 \cdot s \cdot \left(\frac{4}{3}\right)^n$.
- Simplifying some very long equations, we can derive this statement:
  The area of the Koch Snowflake is $\frac{8}{5}$ of the original equilateral triangle.
- The fractal dimension of the Koch curve is estimated to be 1.26186.

Mandelbrot Set

- It was first drawn by Robert W. brooks and is a set of complicated numbers.
- The Mandelbrot set repeating a function on a complicated plane.
- There aren't any perimeter for this fractal. In fact, the perimeter of this shape is infinite
- Unlike the perimeter, there is a limited area for this shape. Even though mathematises haven't found the exact number, it is estimated to be 1.506484 square units.



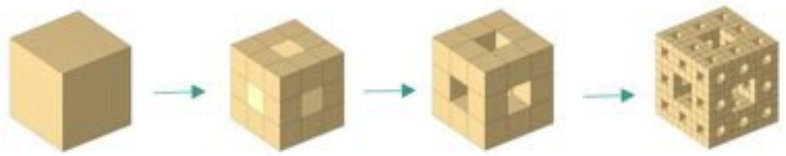
Picture of Mandelbrot when it was first published.



Picture of Mandelbrot set

Menger Sponge

- The Menger Sponge is a 3D fractal.
- The Menger Sponge is constructed in the following way:
  - Begin with a cube.
  - Divide each face of the cube into nine squares. Therefore, the cube gets divided into 27 new smaller cubes.
  - Remove the larger cube in the center of the big cube and the smaller cube in the middle of each face, leaving 20 cubes. The remaining structure is the first iteration.
  - For each of the remaining smaller cubes, repeat steps 2 and 3 and keep iterating.
- Using some mathematical formulas and observations of the Menger Sponge, we can conclude that the volume of the Menger Sponge is $(\frac{20}{27})^n$.
- The fractal dimension of the Moenger Sponge is around 2.73.

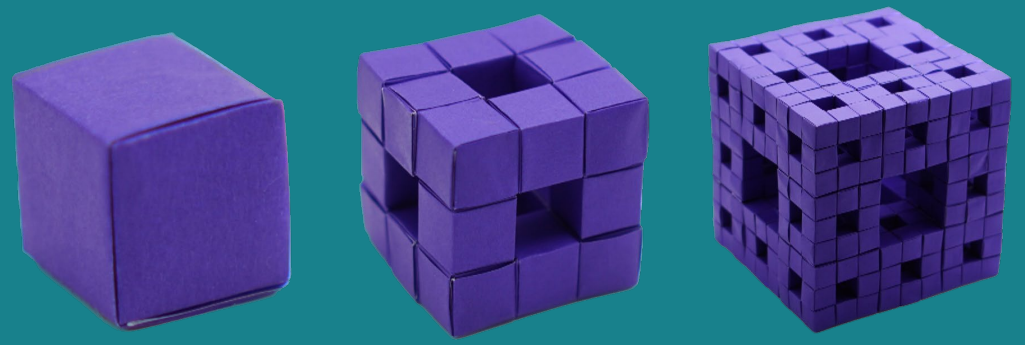


Math Problems

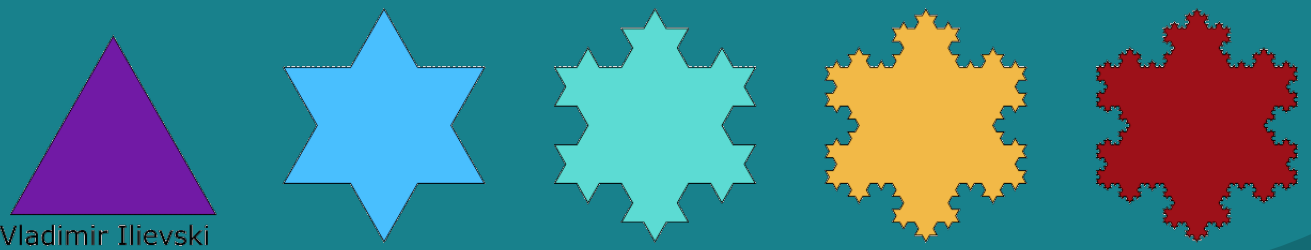
- What is a fractal in math?
- What fraction of the area of the whole triangle are smaller triangles which make up the first stage of the Sierpinski triangle? What about the second stage?

Math Problems (contd.)

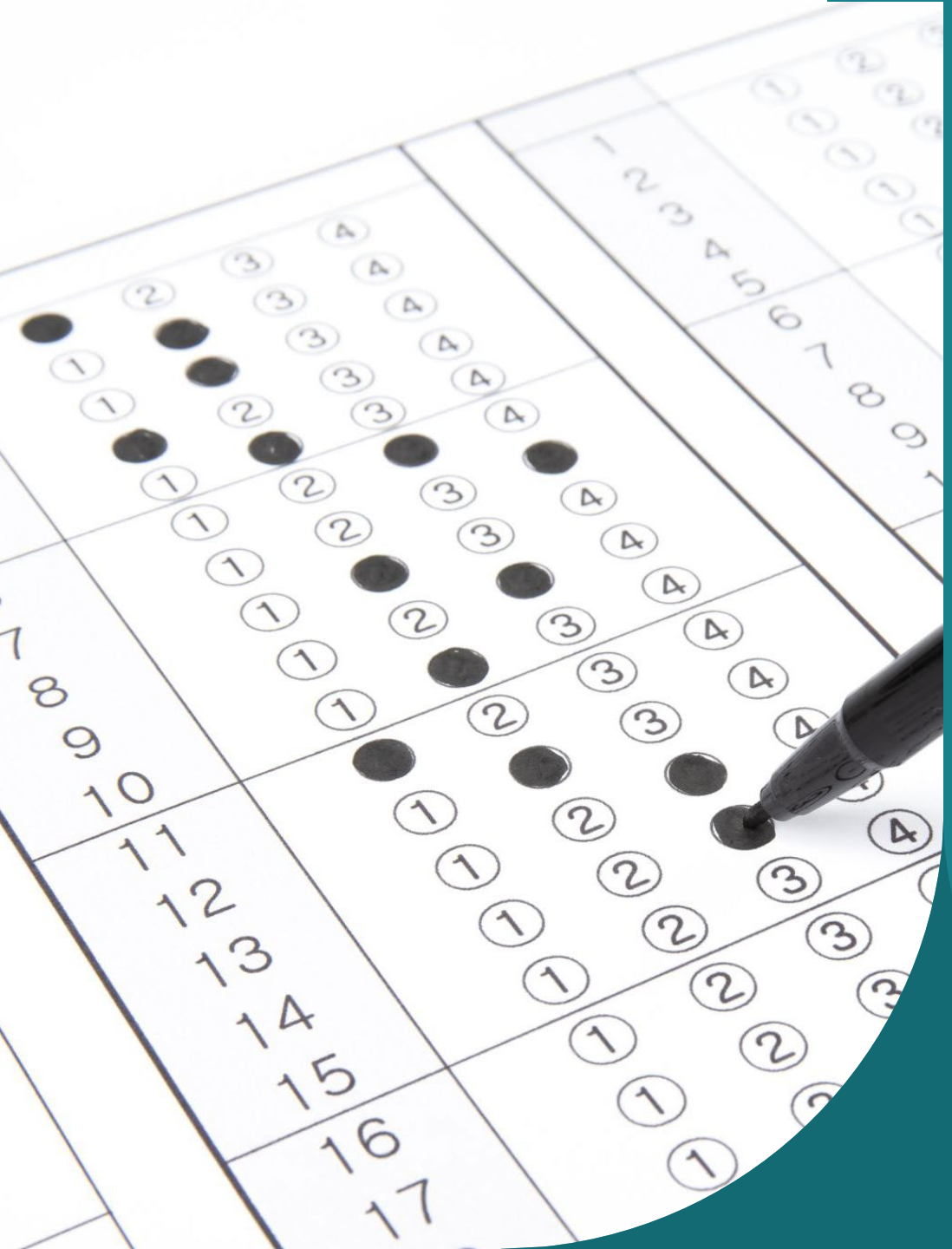
What fraction of the volume of the Menger Sponge is removed to go from stage 0 to stage 1?



How many triangles are added to the curve when you go from stage (2) to stage (3) in the Koch Snowflake?



by Vladimir Ilievski



The answers

1. A fractal is a mathematical shape that is infinitely complex.
2. 1/4, 7/16
3. 7/20
4. 6

Conclusion

- 2D fractals
  - Sierpinski Triangle
  - Koch Snowflake
  - Mandelbrot Set
- 3D fractals
  - Menger Sponge


Thank You