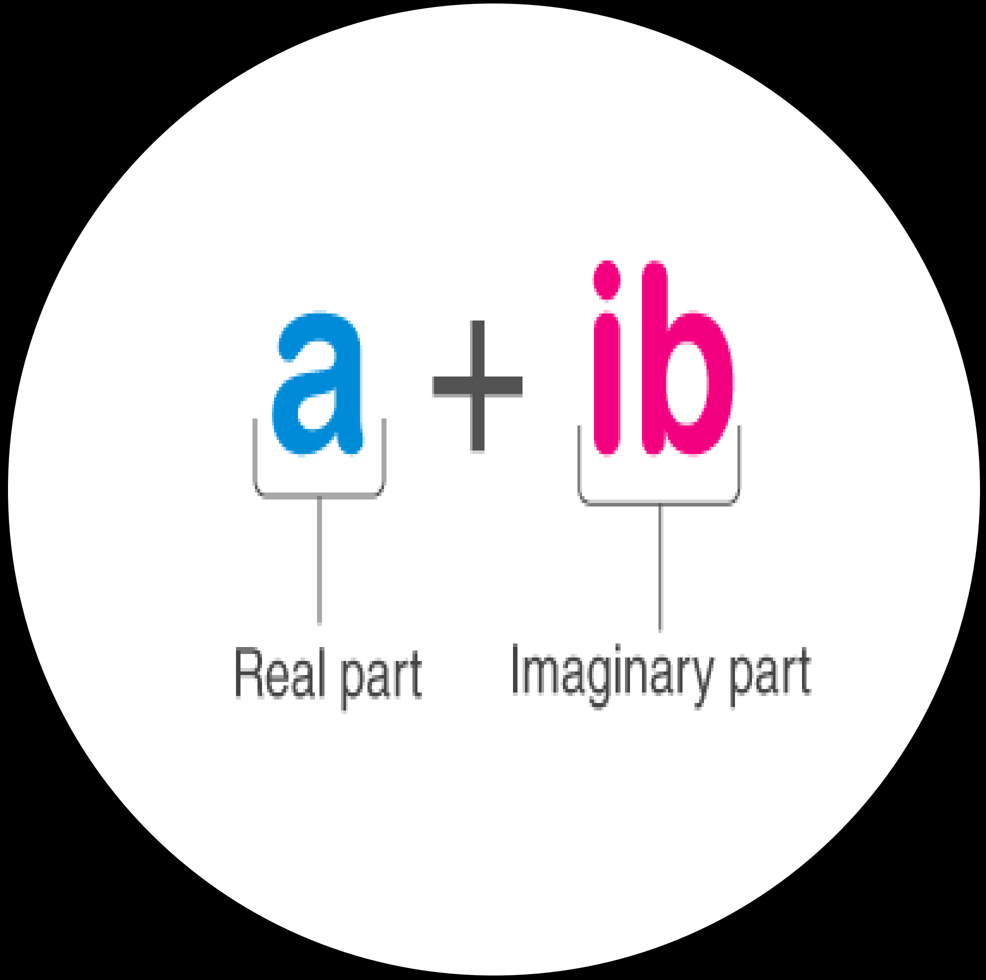
COMPLEX NUMBERS

By Jerry And Lauren

WHAT ARE COMPLEX NUMBERS?

- A complex number is an element of a number system that extends real numbers with an element called i, where $i^2 = -1$.
- Complex numbers are numbers in the form of a+bi, where a and b are real numbers, and i is imaginary. For example, 3+5i.
- Complex numbers can also be visually represented as a pair of numbers (*a*, *b*) forming a vector on a diagram called an Argand diagram, representing the complex plane . *Re* is the real axis, *Im* is the imaginary axis, and *i* is the "imaginary unit", that satisfies $i^2 = -1$.



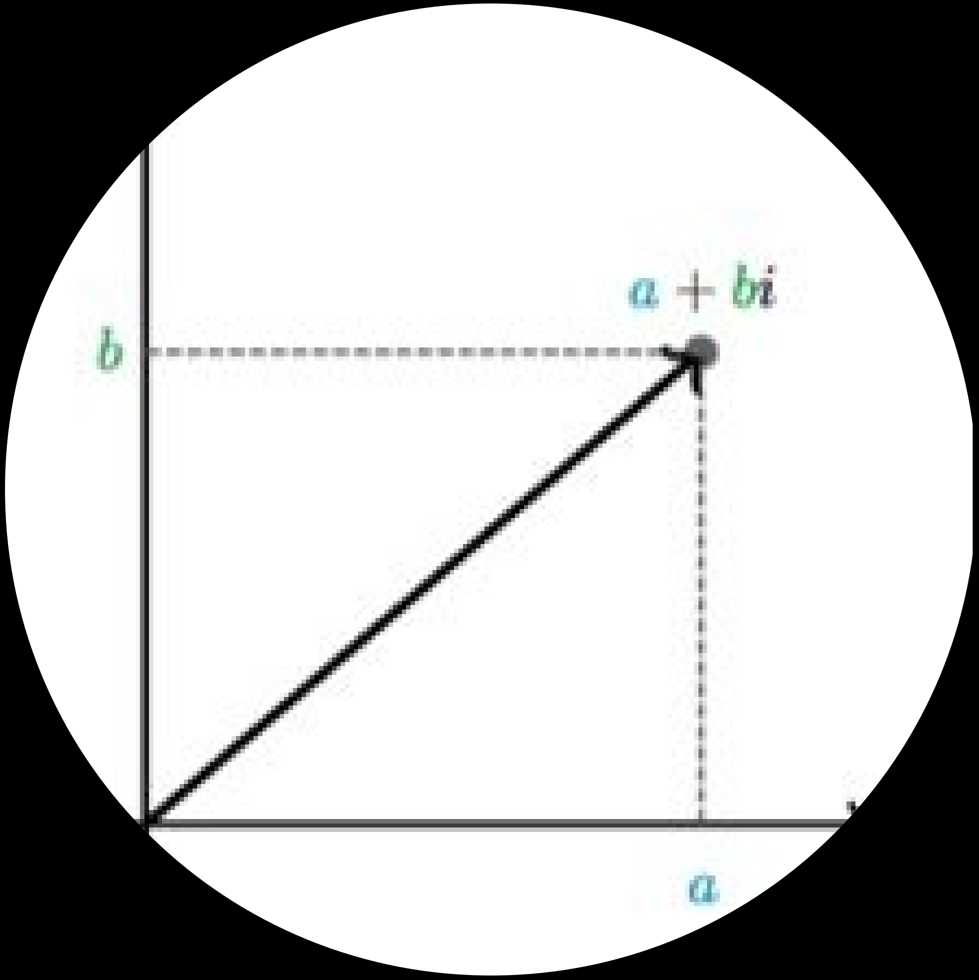
HISTORY OF COMPLEX NUMBERS

- Gerolamo Cardano conceived of complex numbers in around 1545 in his Ars Magna though his understanding was rudimentary; moreover, he later dismissed complex numbers as "subtle as they are useless".
- Many mathematicians contributed to the development of complex numbers. The rules for addition, subtraction, multiplication, and root extraction of complex numbers were developed by the Italian mathematician Rafael Bombelli. A more abstract formalism for the complex numbers was further developed by the Irish mathematician William Rowan Hamilton.

Gerolamo Cardano

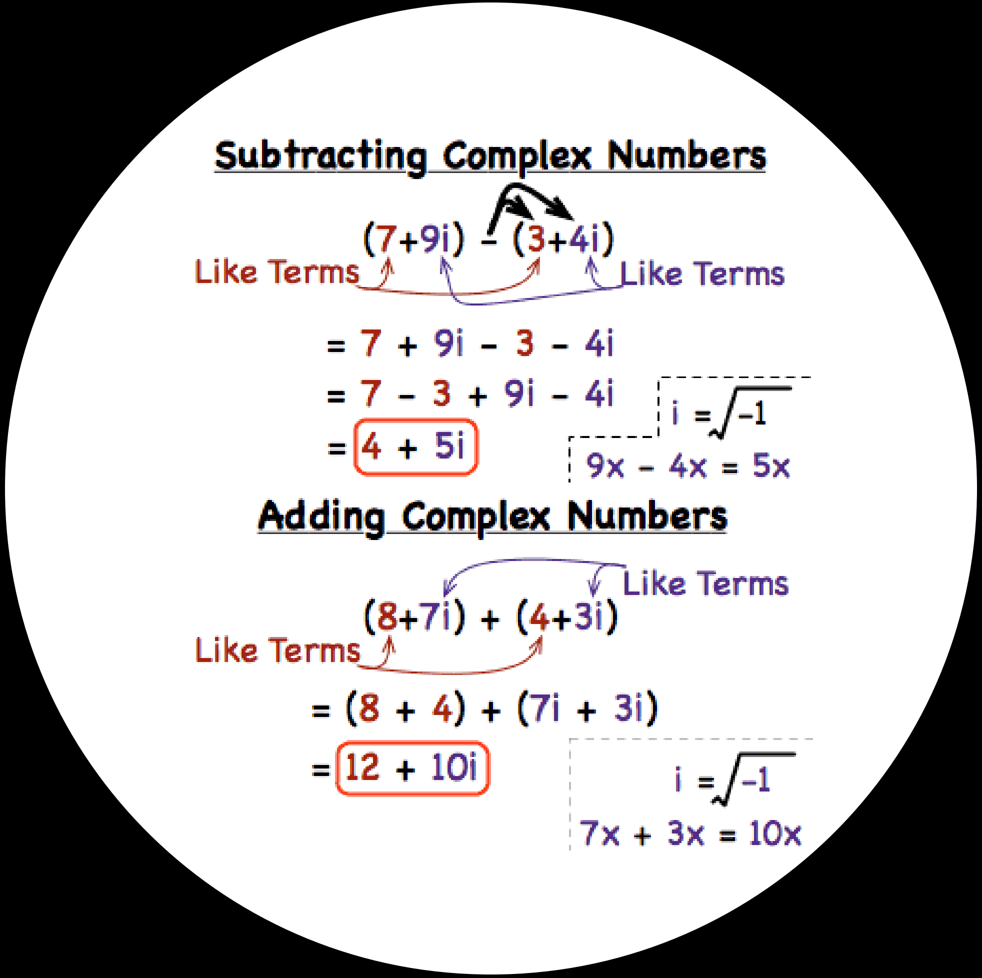
VISUALIZATION OF COMPLEX NUMBERS

- The definition of the complex numbers involving two arbitrary real values immediately suggests the use of Cartesian coordinates in the complex plane.
- The horizontal (*real*) axis is generally used to display the real part, with increasing values to the right, and the imaginary part marks the vertical (*imaginary*) axis, with increasing values upwards.



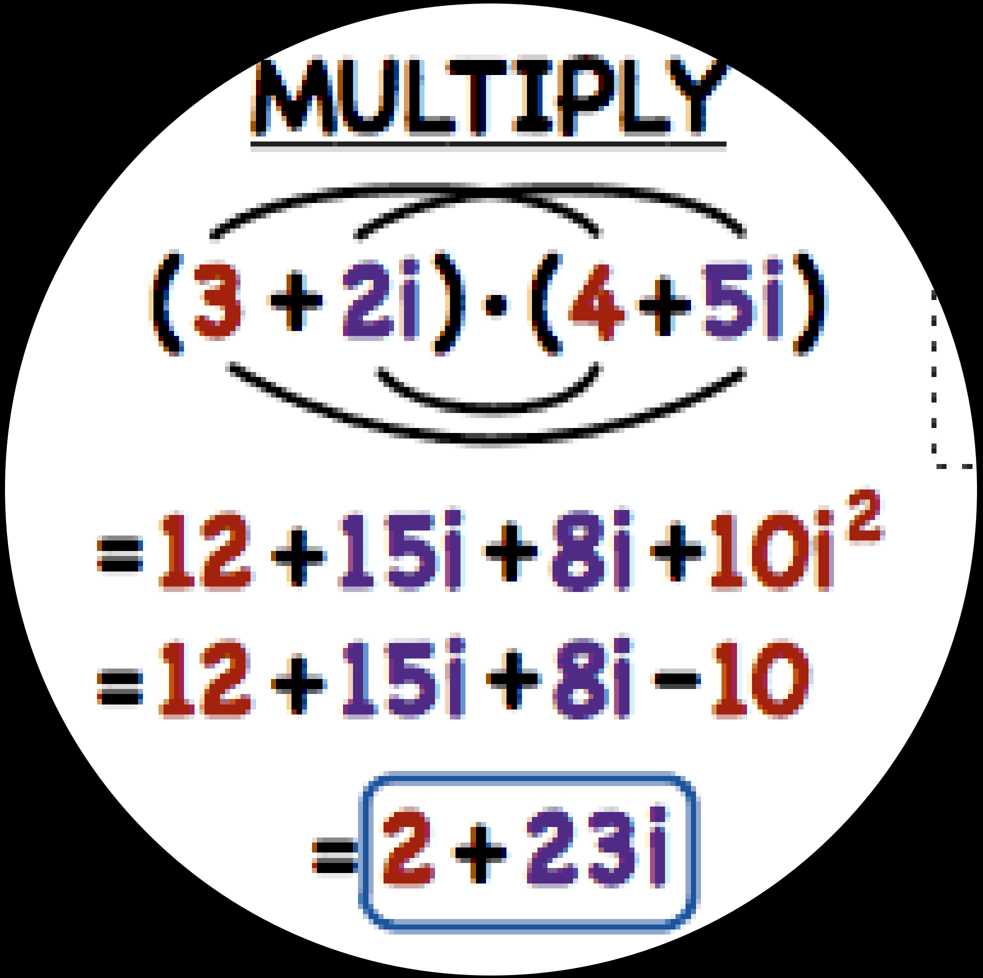
ADDING AND SUBTRACTING COMPLEX NUMBERS

- Adding and subtracting complex is basically like adding and subtracting variables and numbers. The imaginary part should be added together, and the numbers should be added together.
- For example, (a+bi)+(c+di)=a+c+bi+di
- Also, (a+bi)-(c+di)=a-c+bi-di.



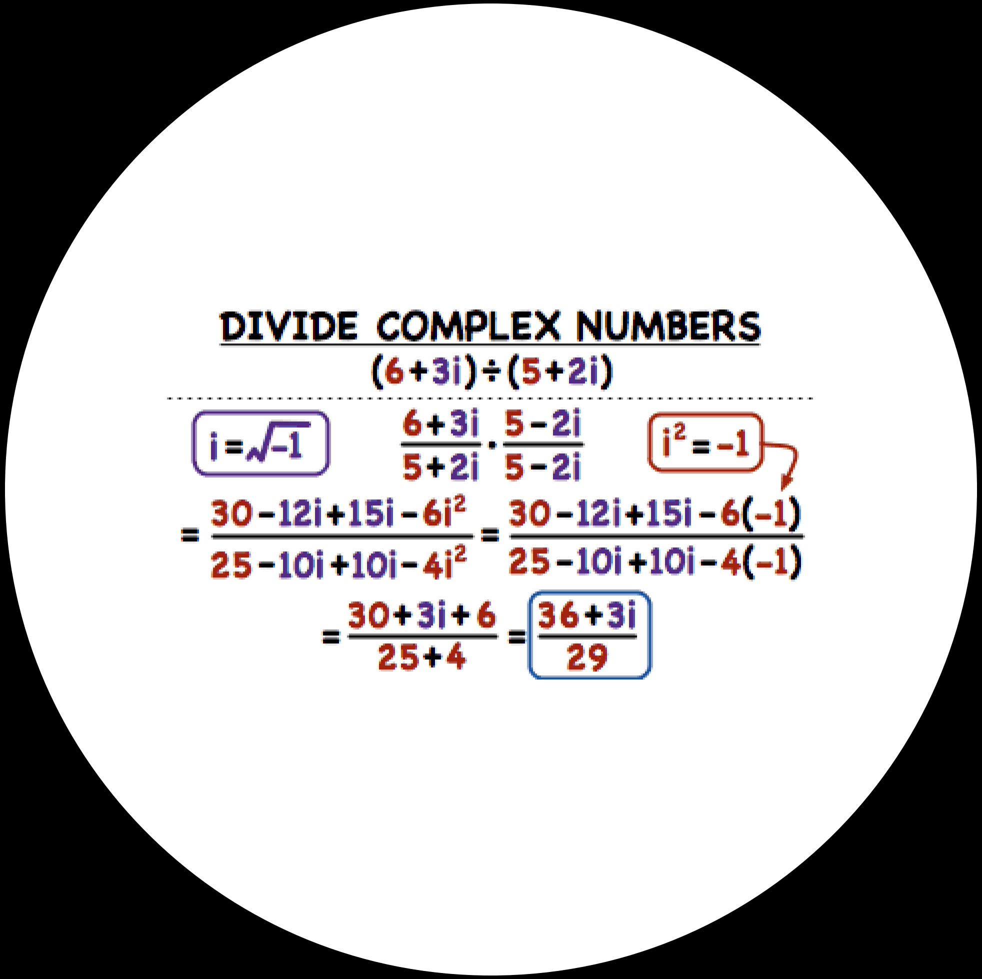
MULTIPLYING COMPLEX NUMBERS

- When multiplying complex numbers, you should multiply each term by each of the other terms in the other bracket.
- Also, you can multiply complex numbers by using this formula: $(a+bi)(c+di)= ac-bd+(c+ad)i$



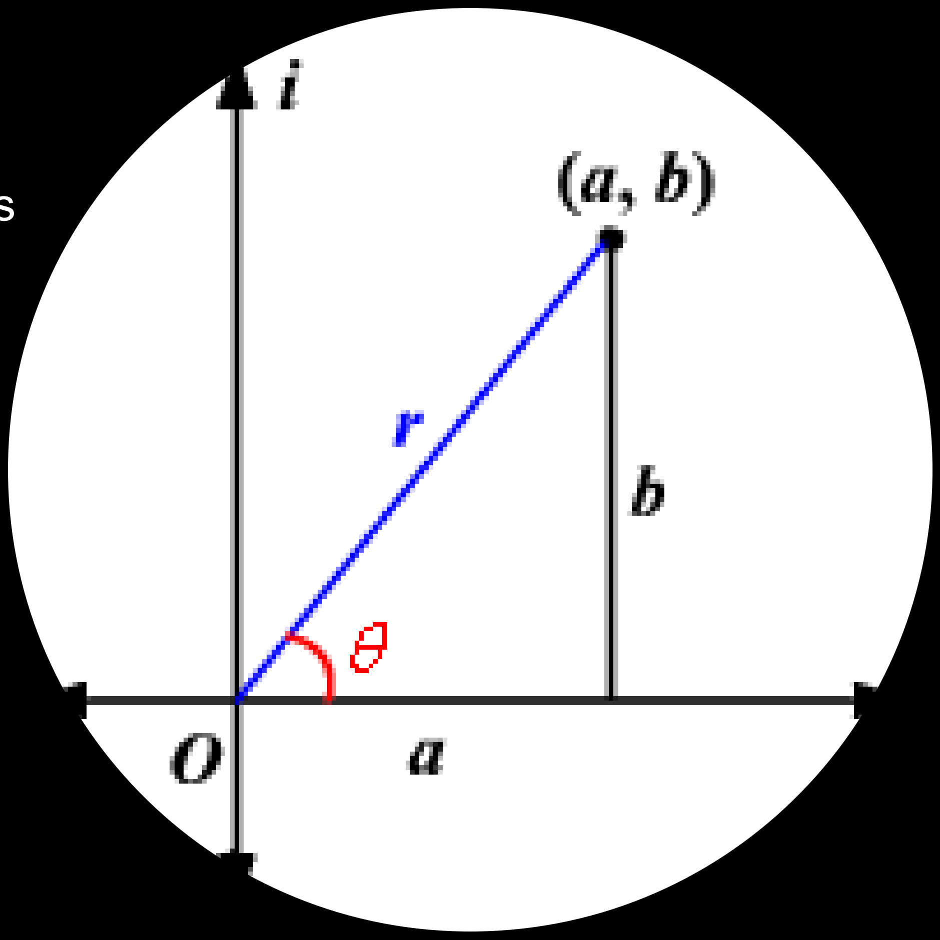
DIVIDING COMPLEX NUMBERS

- When dividing complex numbers, you should multiply the numerator and denominator by the denominators conjugate.
- The equation is: a+bi/c+di=(a+bi)(c-di)/(c+di)(c-di)



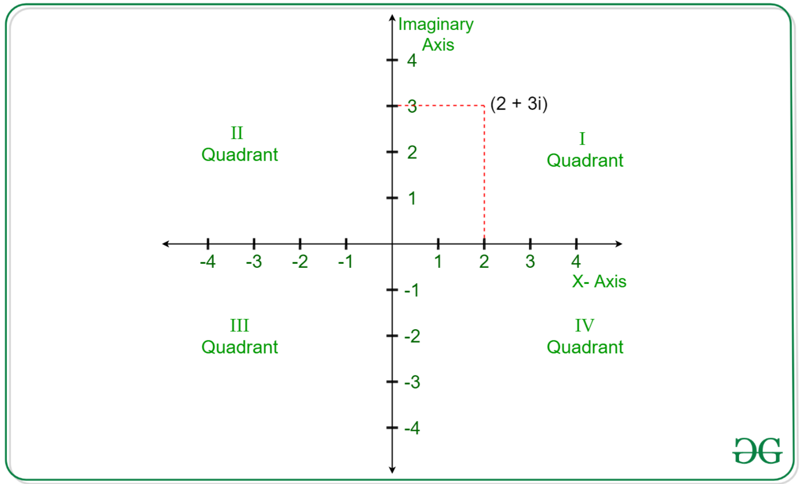
POLAR FORM

- The polar form is another way of representing complex numbers.
- The horizontal axis is the real axis and the vertical axis is the imaginary axis.
- The real and complex components are found using terms r and θ. R is the length of the vector and θ is the angle created by the vector and the horizontal axis.
- The pythagorean theorem demonstrates $r^2=a^2+b^2$. By using trigonometry ratios $\sin\theta=b/r$ and $\cos\theta=a/r$. If we multiply both by r, we get $r(\sin\theta)= b$ and $r(\cos\theta)= a$. If the values are substituted for a and b, we figure out that the polar form of a complex number is $z= (\cos\theta+i\sin\theta)$.
- Regarding complex numbers, r represents the absolute value and the angle θ is the argument of the complex number.



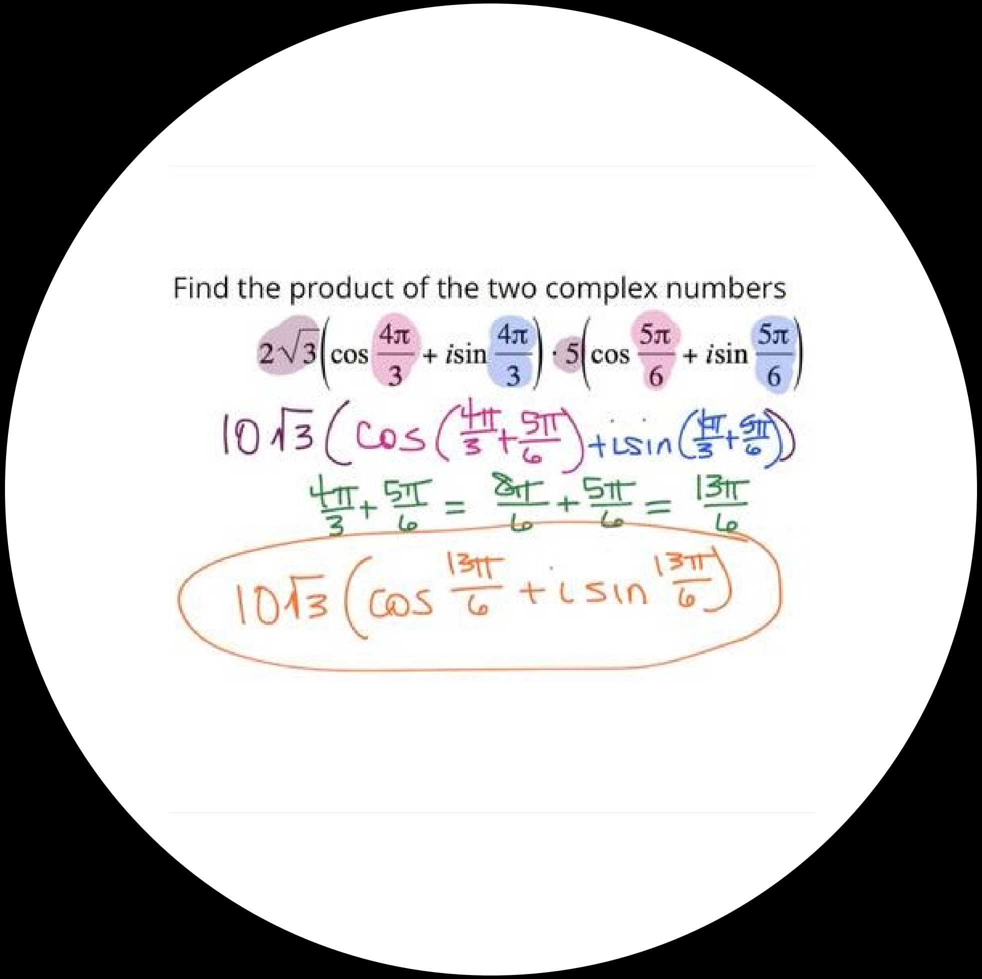
REPRESENTATION OF POLAR FORM

- Complex numbers are written of distance from the origin and an angle from the positive horizontal axis
- $\theta = \tan^{-1}(b / a)$ if z lies in the first or fourth quadrant
- $\theta = \tan^{-1}(b / a) + 180°$ if z lies in the second quadrant
- $\theta = \tan^{-1}(b / a) - 180°$ if z lies in the third quadrant
- Conversion from rectangular form to polar form of complex numbers: $r = \sqrt{(a^2 + b^2)}$



PRODUCT OF POLAR FORM OF COMPLEX NUMBERS

- $z = r_1(\cos\theta_1 + i\sin\theta_1)$, $w = r_2(\cos\theta_2 + i\sin\theta_2)$
- Multiply the coefficients of the complex numbers
- Add θ_1 and θ_2
- Substitute the values to obtain product of complex numbers
- $zw = r_1 r_2\ [\cos(\theta_1 + \theta 2) + i\sin(\theta_1 + \theta_2)]$



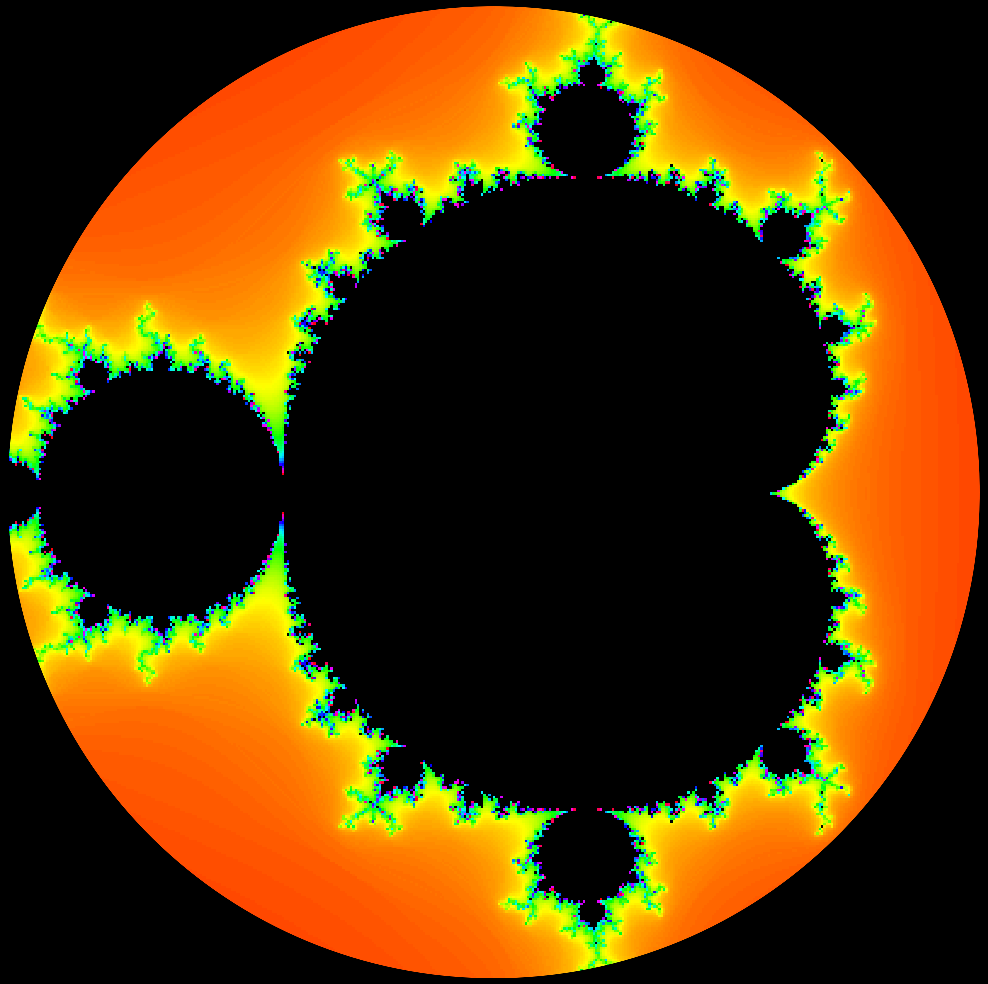
ORDERING COMPLEX NUMBERS AND EQUALITY OF COMPLEX NUMBERS

- There is no natural ordering of complex numbers because complex numbers do not have the structure of an ordered field and is not compatible with addition and multiplication and the non trivial sum of squares in an ordered field is not zero.
- The lexicographic order is incorrect because it is useless in defining completeness
- The well ordering principle does not apply in complex numbers
- Two complex numbers are equal if both their real parts and imaginary parts are equal

$$a_1 + ib_1 = a_2 + ib_2 \iff a_1 = a_2 \wedge b_1 = b_2$$

MANDELBROT SET

- The mandelbrot set is based on complex numbers, illustrating the equation ($z_{n+1}=z_n^2+c$).
- The mandelbrot set was named after mathematician Benoit B. Mandelbrot
- The mandelbrot set will continue until it diverges to infinity, where a color is chosen based on how quickly it diverges or does not divulge and forms the actual mandelbrot set, represented as black.



PRACTICE PROBLEMS

- Problem 1

  What is (3i-5)(5i-3)?

  A) 34   B) -34i   C) 34i   D) 30-34i

- Problem 2

  What is (8i-1)/(3i-9)?

  A) (11-23i)/30   B) (11+23i)/30   C) 17/15   D) ¾

- Problem 3

  What is (-2i-4)+(7i+6)-(9i-1)?

  A) 4-3i   B) 7   C) 3-4i   D) -1

ANSWERS TO THE PRACTICE PROBLEMS

- The answer to Problem 1 is B.
- The answer to Problem 2 is A.
- The answer to Problem 3 is C.

PRACTICE PROBLEMS

Problem 4

What complex in polar form represents the complex number $z=4+3i$?

A)$Z=5(\cos(36.9)+i\sin(36.9))$ B)$7(\cos(51.3)+i\sin(51.3))$ C)$5(\cos(0.01)+i\sin(0.01))$ D)$25(\cos(36.9)+i\sin(36.9))$

Problem 5

What is the argument and absolute value of the complex number $z=5+12i$?

A)$R=13,\ \theta=22.6$ B)$R=13,\ \theta=67.4$ C)$R=169,\ \theta=67.4$ D) $R=17,\ \theta=22.6$

Problem 6

What is $5(\cos(pi/2)+i\sin(pi/2)) \times 2(\cos(pi/3)+i\sin(pi/3))$?

A)$7(\cos(5pi/6)+i\sin(5pi/6))$ B)$10(\cos(5pi/6)+i\sin(5pi/6))$ C)$10(\cos(pi^2/6)+i\sin(pi^2/6))$ D)$25(\cos(pi^2/6)+i\sin(pi^2/6))$

ANSWERS TO THE PRACTICE PROBLEMS

Problem 4

$R=\sqrt{3^2+4^2}=5$

$\theta=\tan^{-1}(3/4)=36.9$

$Z=5(\cos(36.9)+i\sin(39.6))$

Problem 5

$R=\sqrt{5^2+12^2}=13$

$\theta=\tan^{-1}(12/5)=67.4$

Problem 6

5x2=10

pi/2+pi/3=5pi/6

z=10(cos(5pi/6)+isin(5pi/6)).

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CONCLUSION

In this presentation, we talked about what complex numbers are, the history of complex numbers, visualization of complex numbers, adding and subtracting complex numbers, multiplying and dividing complex numbers. We have also talked about the polar form of complex numbers, ordering of complex numbers and the mandelbrot set. The mandelbrot set is an illustration of complex numbers. Complex numbers cannot be ordered. The polar form is another way of representing complex numbers.

THANK YOU

FOR WATCHING!

CREDITS

- Wikepidia
- Virtual Nerd- Adding Complex Numbers
- Virtual Nerd- Subtracting Complex Numbers
- Study.Com
- Cuemath-Polar Form
- Study.com-Multiplication of complex numbers in polar form