Algebraic Geometry

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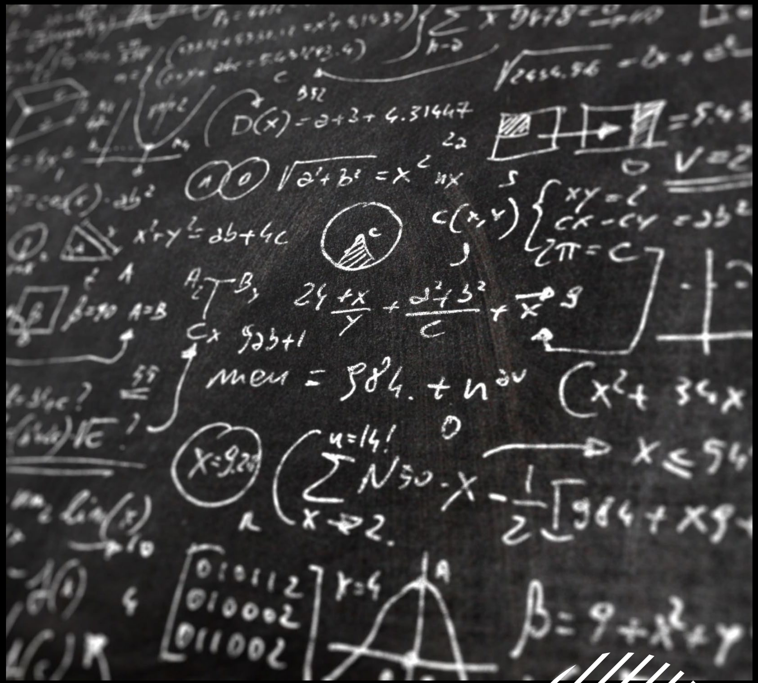
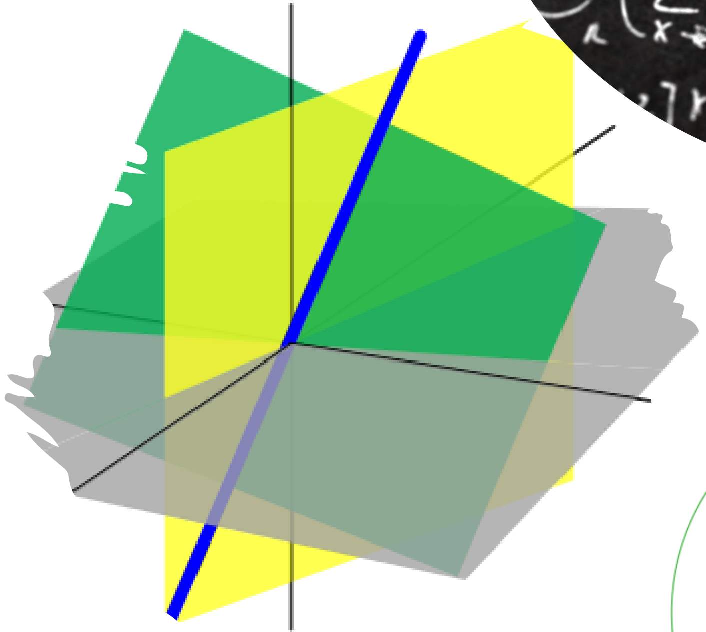
What is Algebraic Geometry?

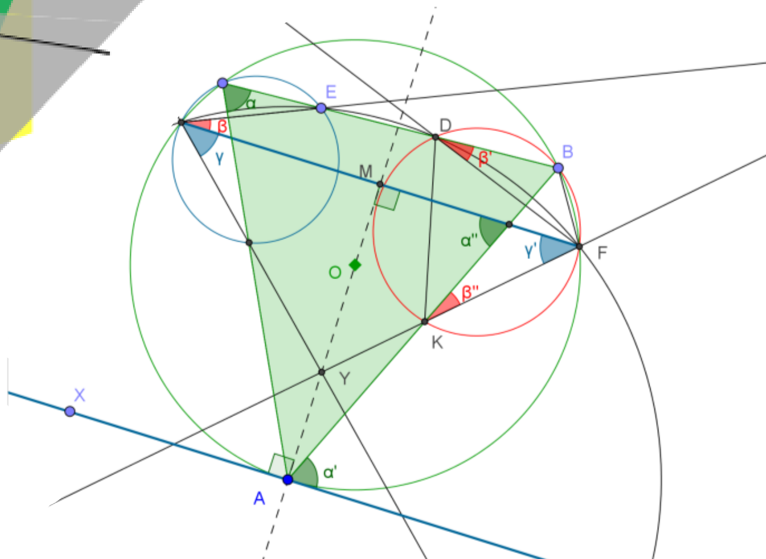


Algebraic geometry helps solve geometry problems by using algebraic equations and formula.

Branches of Algebraic Geometry

There are four types of algebraic geometry: **real algebraic geometry, algebraically closed field, Diophantine geometry,** and **analytic geometry**. In this presentation we are going to tell something about each topic.





Solving Algebraic Geometry

We also are going to give you an algebraic geometry quiz that you can solve if you remember the formulas that we are going to show you!

Pay attention!

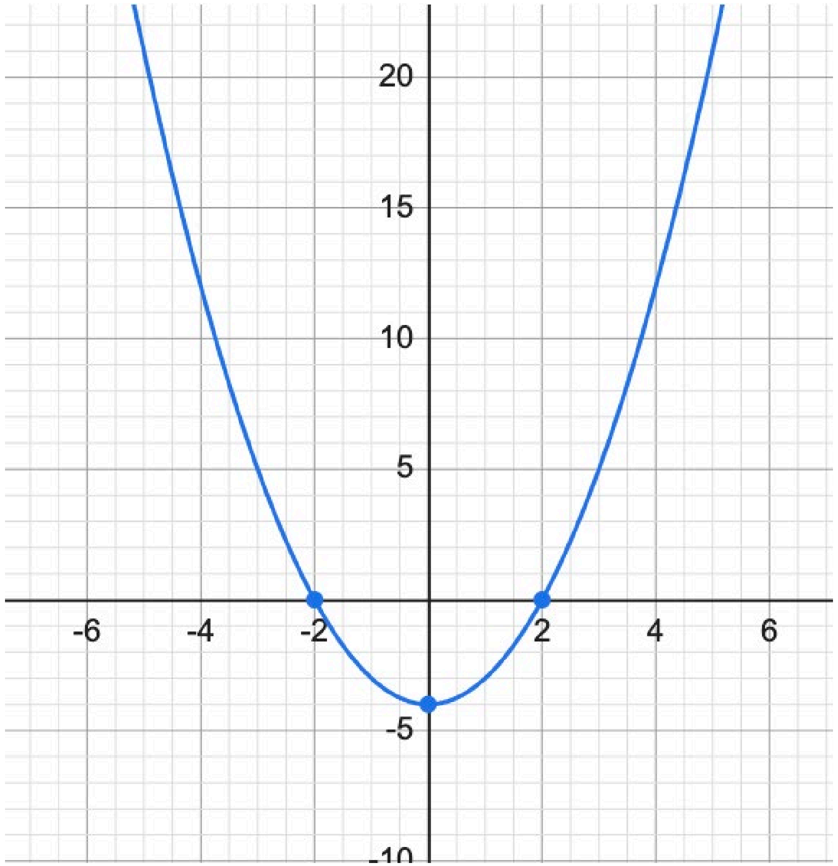
Real Algebraic Geometry

Real algebraic geometry is a branch of algebraic geometry. It is used to analyze geometry of algebraic sets and to develop new solutions to hard non-convex problems. Computational real algebraic geometry is used to figure out algorithmic aspects to algebraic geometry.

Consider the polynomial equation $f(x) = x^2 - 4 = 0$

The solution of this equation is $x = \pm 2$.

This equation also represents a parabola opening upwards.



Algebraically Closed Field

How can a type of math be closed???
Well, it's not actually closed. It's just a category of numbers.

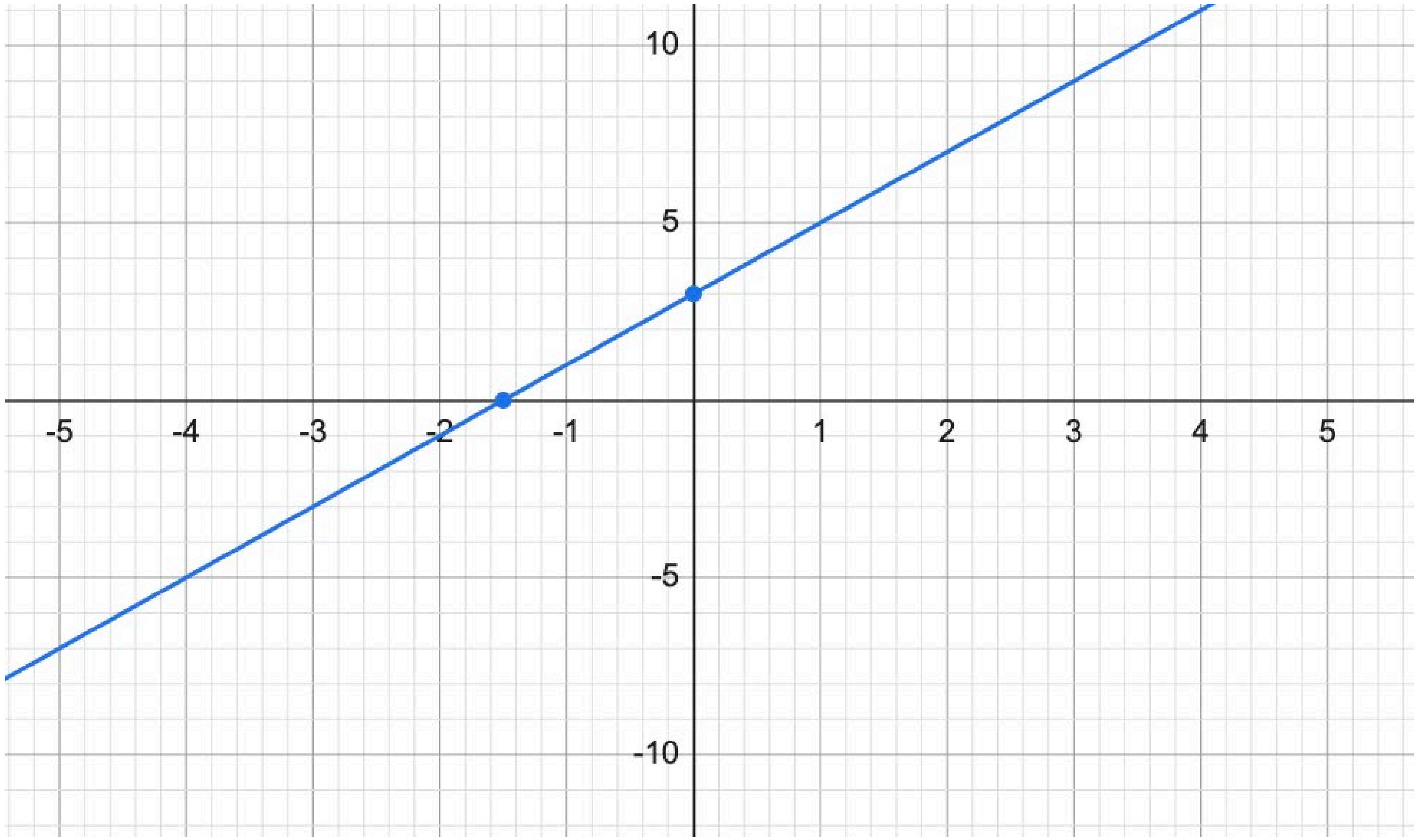
Algebraically Closed Field is a field in which the numbers in an equation are a root of another number in the equation. Let's consider $x^2 + 1 = 0$. There's no real number x that satisfies the equation but in the field of complex numbers, we can find solutions.

Algebraic Closed Field also studies the properties telling if an equation is algebraically closed or not.

Analytic Geometry

Analytic geometry is the study of geometry using a coordinate system. This type of geometry is used in physics and engineering, and it is the foundation of algebraic geometry.

For example, consider y = 2x + 3. This equation represents a straight line with a slope of 2 and a y-intercept of 3.



Diophantine Geometry

Diophantine geometry is the study of Diophantine equations which is used a lot in algebraic geometry. In Diophantine geometry four theorems are used to solve equations. These theorems solve equations where there are two or more variables.

The four Diophantine equations

1. Thue's Theorem
2. Siegel's Theorem
3. Roth's Theorem
4. Subspace Theorem

These Theorems play a crucial role in understanding the distribution and density of rational solutions to Diophantine equations.

Now we're going to show you how to solve Algebraic Geometry problems!

$$(x-h)^2+(y-k)^2=r^2$$

A useful formula is this one. You can use it to calculate the radius of the circle.

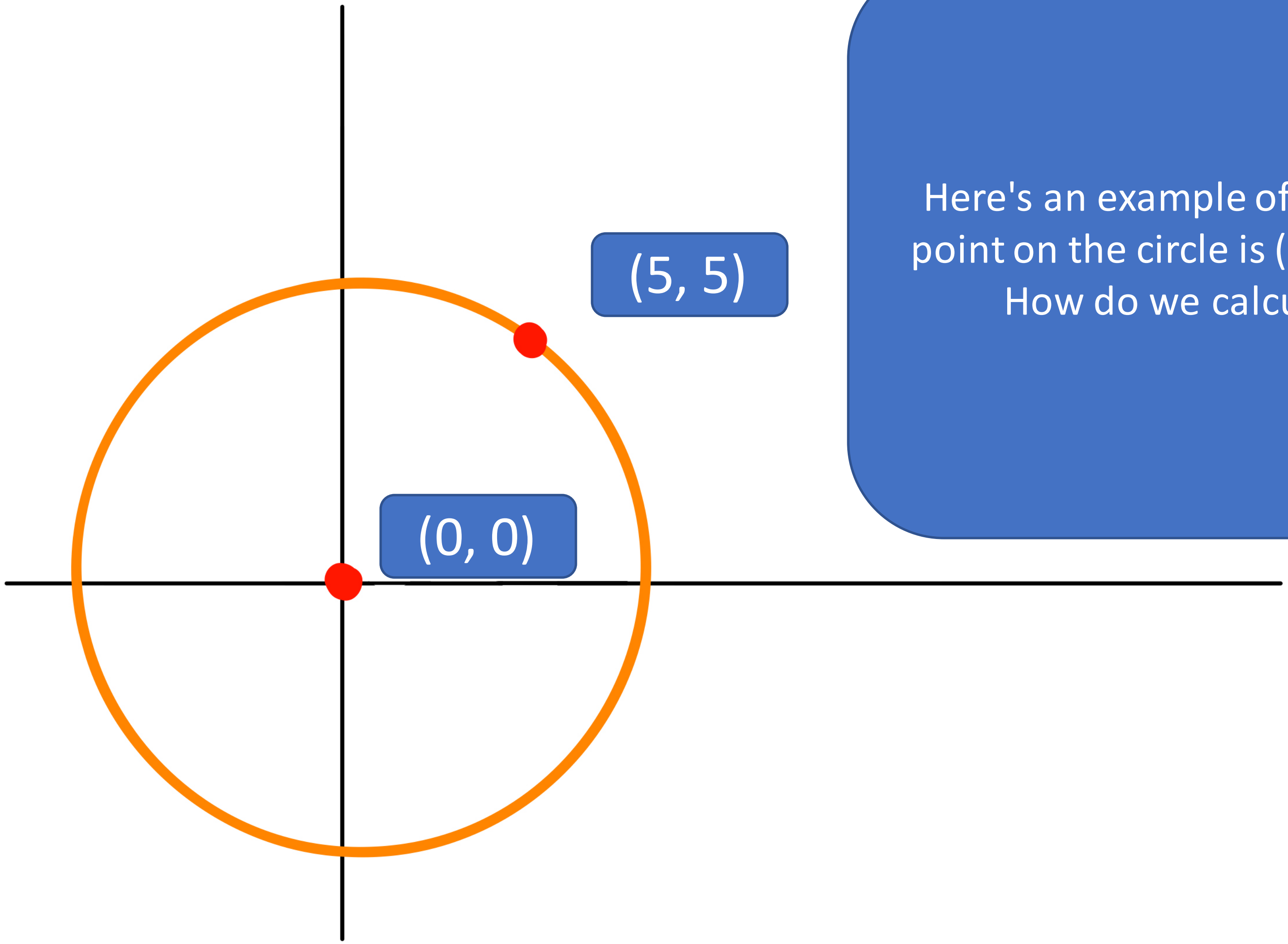
x = the x coordinate of a point on the circle

y = the y coordinate of the same point on the circle

h = x coordinate of the center point of the circle

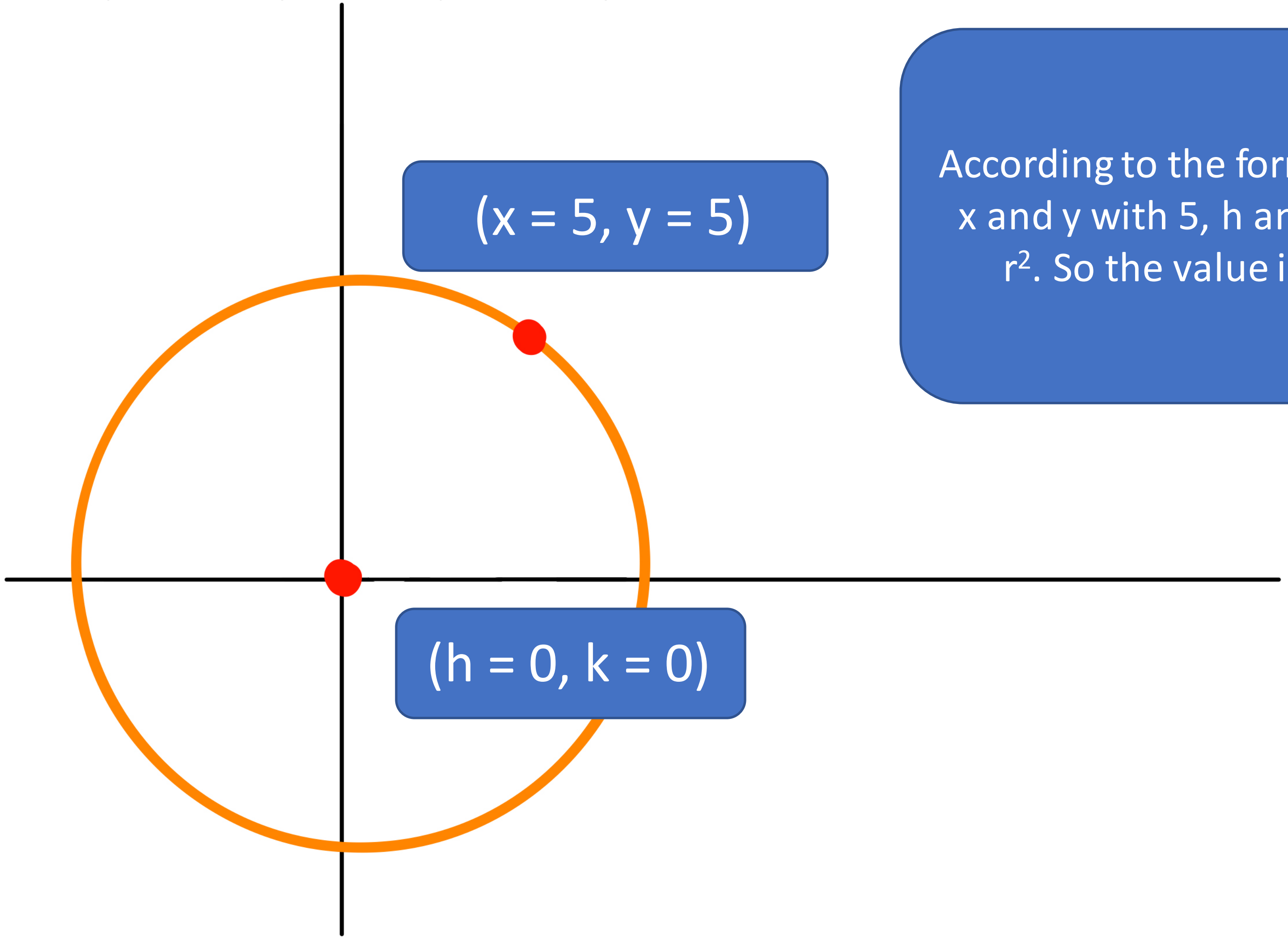
k = y coordinate of the center point of the circle

Formula #1



Here's an example of the formula. The coordinate of a point on the circle is (5, 5) and the center point is (0, 0). How do we calculate the radius of the circle?

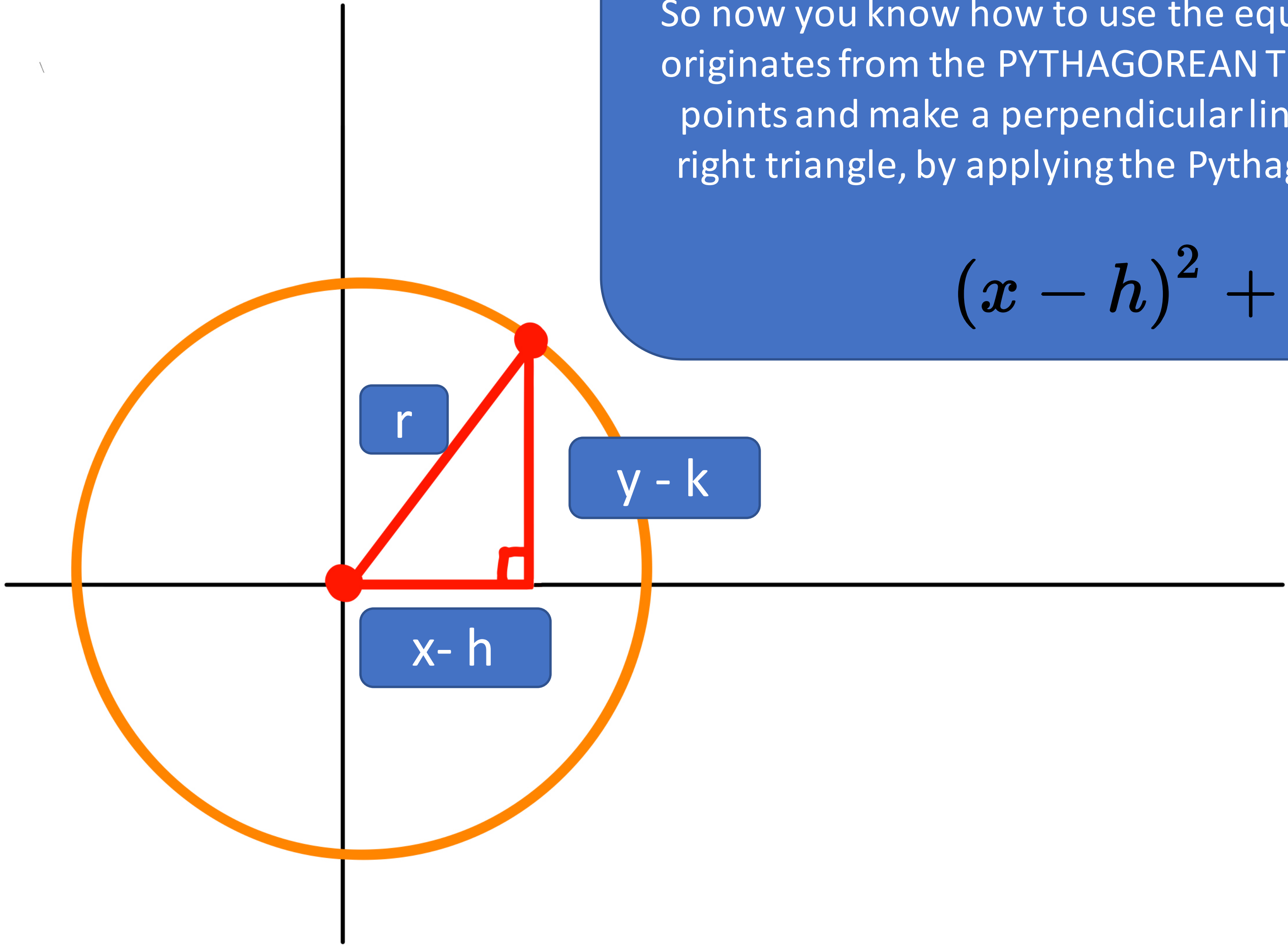
$$(x-h)^2+(y-k)^2=r^2$$



According to the formula, and when you replace the value x and y with 5, h and k with 0, you get $(5 - 0)^2 + (5 - 0)^2 = r^2$. So the value is $25 + 25 = r^2$ so $r^2 = 50$ and $r = 5\sqrt{2}$

So now you know how to use the equation, but where did it come from? It originates from the PYTHAGOREAN THEOREM! When you connect the two points and make a perpendicular line from the point to axis x, it makes a right triangle, by applying the Pythagorean Theorem we get the formula.

$$(x-h)^2+(y-k)^2=r^2$$

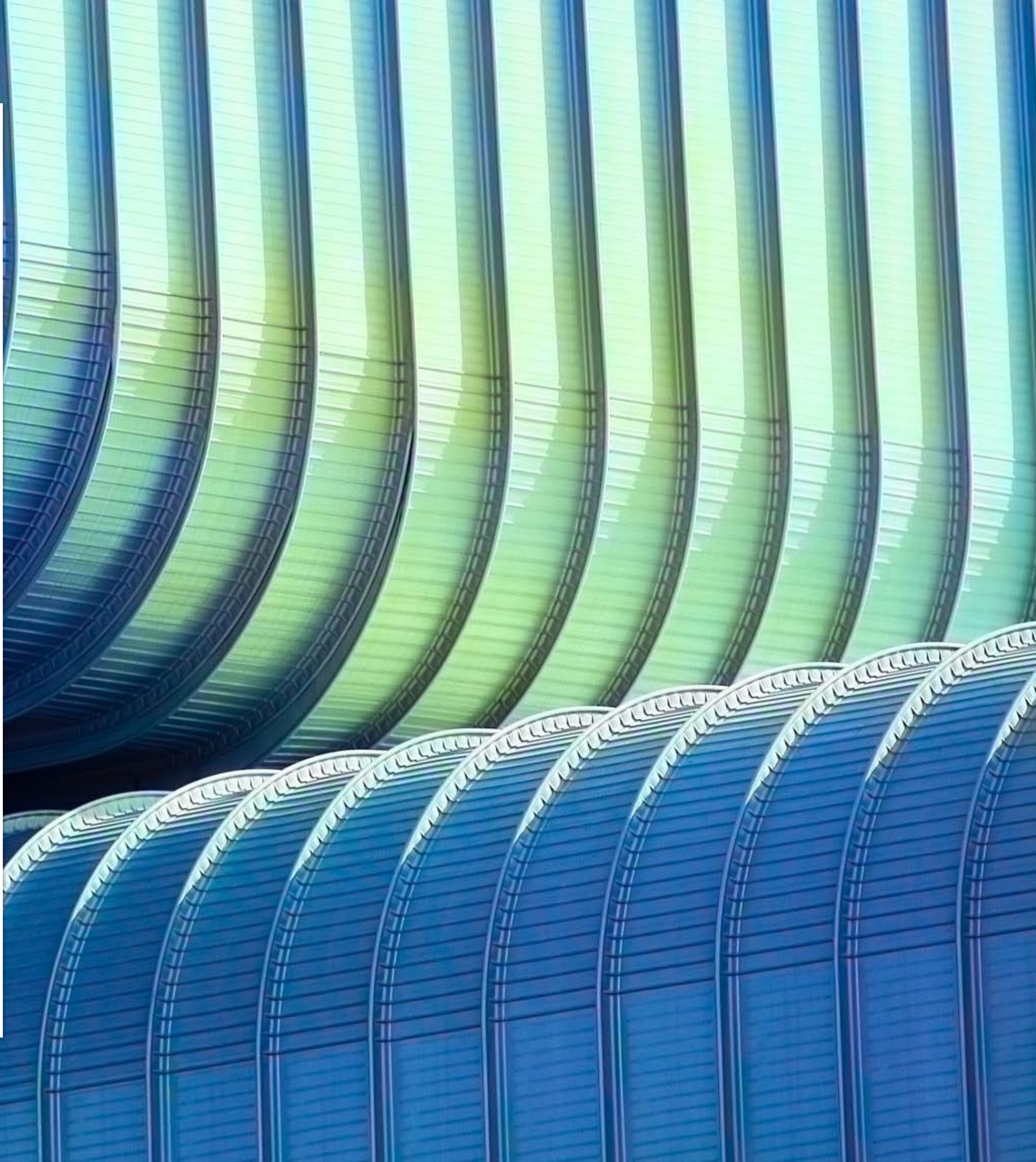


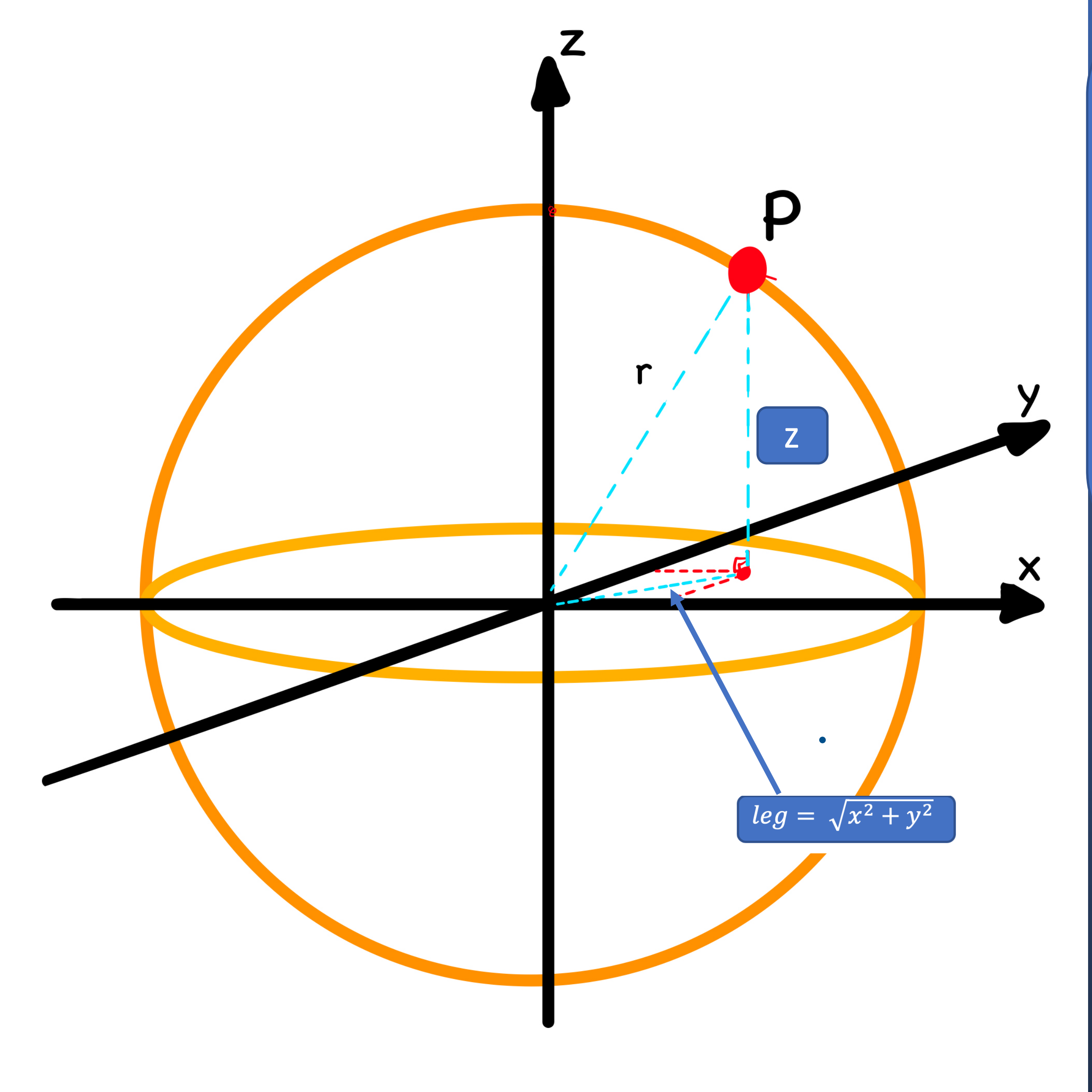
Formula # 2

If you want to know the topmost point of a sphere with radius r you can use an algebraic geometry formula to figure out the point.

This formula is $x^2 + y^2 + z^2 - r^2 = 0$

If you know two points in this formula you can easily solve the topmost point of this sphere





Do you notice the right triangle in aqua blue?
According to Pythagorean Theorem,

$$r^2 = z^2 + leg^2$$

The leg is on the plane of (x, y), again according to Pythagorean Theorem,

$$leg^2 = x^2 + y^2$$

Substitute the value of leg we can get:

$$r^2 = x^2 + y^2 + z^2$$

Formula #3
How to solve the quadratic equation using graphing.

We will use the equation $x^2 + 6x+5$ as an example.

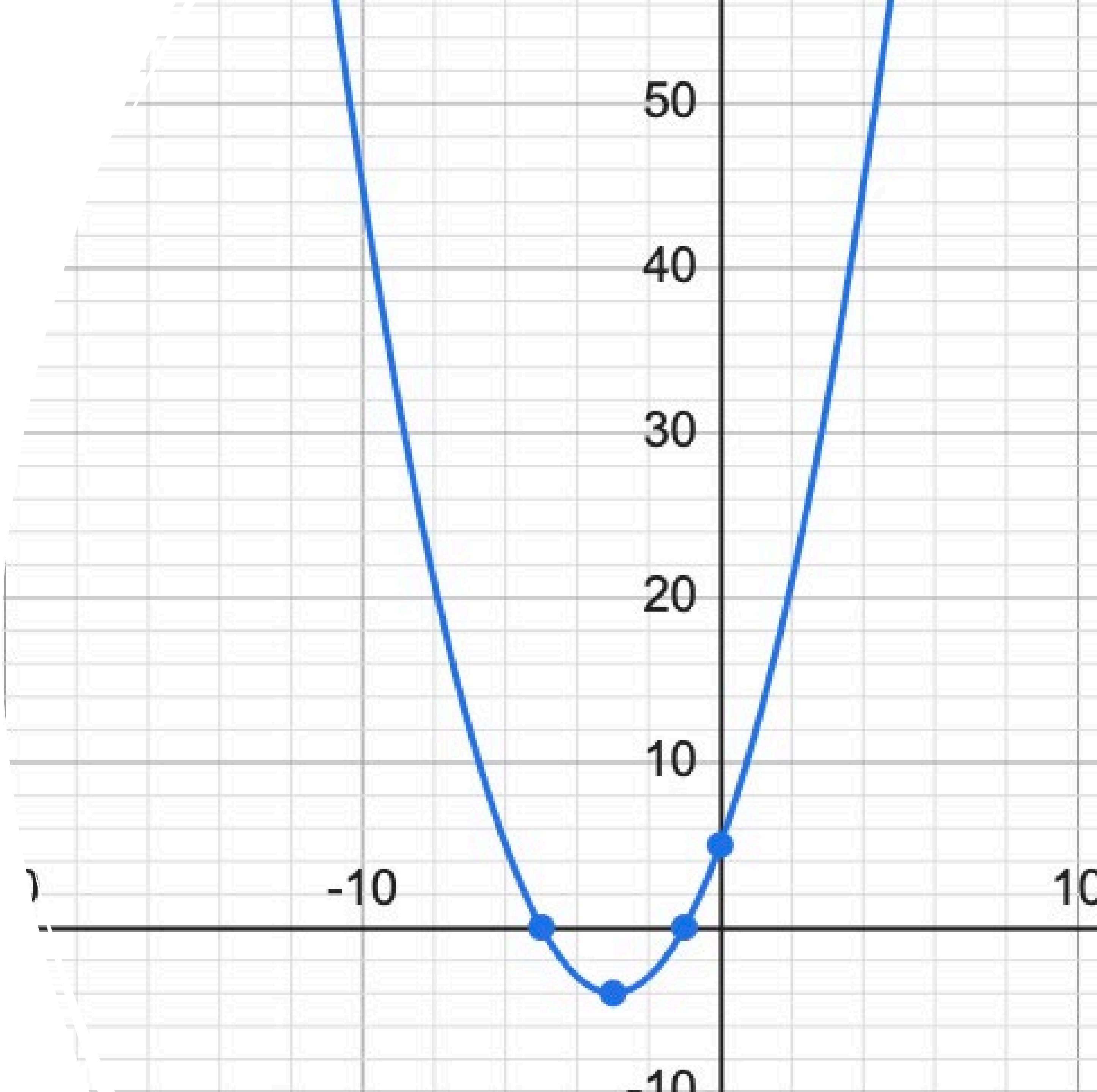
We can start by plugging in a few points.

For example, we can plug in 1 to find that the y coordinate is 12.

When you have plugged in a lot of points, you can connect them, and the x intercepts of the graph are the solutions.

This works because at the x intercept, the y coordinate is 0.

In fact, you can use this to solve any formula using x and y



Formula # 4
A linear equation in two variables
(ax + by + c = 0)

This equation is about a line in two-dimensional affine space can be written in the form $ax + by + c = 0$, where a, b, and c are constants.

The constants a and b determine the slope, while the constant c determines its position in the two-dimensional plane.

If you were to think of it as a ramp, then the slope (a & b) would be how steep the ramp was, and the position (c) would be where the ramp starts and ends

For example, the equation $x – y + 1 = 0$ describes a line that starts on (0,1) and has a slope of one. You can also rewrite the equation so that it is $y = x + 1$ (which is the slope and y-intercept form)

How does this equation relate to Algebraic Geometry?

In algebraic geometry, we study algebraic varieties, which are geometric objects. These geometric objects are defined as the set of solutions to polynomial equations. An example of polynomial equations are the linear equations like the one before. These are special cases of polynomial equations. They correspond to hyperplanes in the space around them and play an important role in algebraic geometry.

Review time… Try the quiz!

https://play.myquiz.org/?_ga=2.195173164.1835030186.1678741479-583410383.1678741477

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Conclusion

Thank you for watching our presentation! We hope we helped you in your mathematical journey!